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Comparative study of several concretes regarding their potential for contributing to prestress losses, May 1968

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Prestress Losses
Progress Report No. 1

**COMPARATIVE STUDY OF SEVERAL CONCRETES
REGARDING THEIR POTENTIALS
FOR CONTRIBUTING TO PRESTRESS LOSSES**

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by
Anoushiravan Rokhsar
Ti Huang

Fritz Engineering Laboratory Report No. 339.1

UNIVERSITY OF ILLINOIS AT CHICAGO
INSTITUTE OF RESEARCH

COMPARATIVE STUDY OF SEVERAL CONCRETES
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FOR CONTRIBUTING TO PRESTRESS LOSSES

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Anoushiravan Rokhsar

Ti Huang

This work was conducted as part of the project Prestress Losses in Pre-tensioned Concrete Structural Members, sponsored by the Pennsylvania Department of Highways, and the U. S. Bureau of Public Roads. The opinions, findings, and conclusions expressed in this report are those of the authors, and not necessarily those of the sponsors.

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May 1968

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ABSTRACT

In this report are presented the results of an experiment conducted to compare the concretes produced at the seven manufacturers of prestressed concrete in Pennsylvania with regard to their potential in contributing to prestress losses. This experiment is the preliminary phase of a research project for the establishment of a rational method to evaluate losses in pre-tensioned concrete structural members.

Small concrete specimens were made at each of the producers of prestressed concrete in Pennsylvania. Elastic shortening and creep strains were measured for an initial prestress of 2000 psi. Shrinkage strains were also measured from specimens of identical dimensions. The two manufacturers whose products exhibited the largest and the smallest elastic shortening, creep, and shrinkage were identified.

This report also reviews previous research on the subject of creep and shrinkage, and presents several known theories concerning these phenomena. The data from this experiment was also used to test several creep and shrinkage formulae currently accepted.

1. INTRODUCTION

1.1 Background

The use of prestressed concrete beams in the United States lagged behind that of many European countries. Construction of the first prestressed concrete bridge in this country, Walnut Lane Bridge in Philadelphia, was started in 1949 and completed in 1951. Since that time, the use of prestressed concrete in bridge construction has been expanding rapidly. During the years 1957 to 1960, 2052 prestressed concrete bridges were authorized by the Bureau of Public Roads for construction.¹⁴ As of this writing, more than 12,000 prestressed concrete highway bridges exist in this country. Pennsylvania, having approximately 1900, leads all the states in the nation for the number of prestressed concrete bridges constructed.

One of the major problems involved in the design of prestressed concrete members is the estimation of the loss in the prestress forces. For pre-tensioned members, these losses are attributed to elastic shortening, creep, and shrinkage in concrete, and relaxation in steel. At the time when the prestress is transferred to the concrete, concrete shortens and the prestressed steel shortens with it. Therefore, there is a

decrease of prestress in the steel which is referred to as the loss due to elastic shortening. Shrinkage is the shortening of concrete due to drying and chemical reactions while creep is the continued shortening due to external loads. Creep and shrinkage shortenings of concrete in turn will cause the prestressing tendons to shorten, and consequently, to lose some of its initial prestress. Relaxation in steel actually signifies the decrease of its stress when it is kept under constant strain for a period of time. Because of creep and shrinkage in concrete, however, the prestressed steel is not maintained under a constant strain. Although not strictly correct, the loss of stress in prestressing steel due to the sustained varying strain is commonly referred to as the relaxation loss. Unlike elastic shortening, creep and shrinkage of concrete and the relaxation in steel are long term phenomena and theoretically continue indefinitely.

The present method of estimating prestress losses in pre-tensioned concrete beams vary widely in different specifications. The following formula is recommended by ACI-ASCE Joint Committee¹ and AASHTO:²

$$\Delta f_s = (\epsilon_e + \epsilon_s + \epsilon_c) E_s + \delta_1 f_{si}$$

In this formula Δf_s is the loss in prestress, and f_{si} is the initial stress in steel. The symbols ϵ_e , ϵ_s , and ϵ_c signify the strains in concrete due to elastic shortening, shrinkage, and creep, respectively. The ratio of the relaxation loss to the initial prestress force in steel is δ_1 . The Joint Committee also recommended the use of 35,000 psi total loss in the pre-tensioned beams in the absence of necessary data. The U.S. Bureau of Public Roads recommends a more specific formula,²²

$$\Delta f_s = 6000 + 16 f_{cs} + 0.04 f_{si}$$

where f_{cs} is the initial stress in concrete at the level of centroid of steel. In this equation, the 6000 psi represents the shrinkage loss. The term $16 f_{cs}$ may be divided into $5 f_{cs}$ for elastic shortening, and $11 f_{cs}$ for the creep in concrete. The last term represents the loss due to relaxation in steel.

In the design of standard bridge members of the Pennsylvania Department of Highways, the loss of prestress is estimated as 20% for box girders and 22.8% for I-beams. The Pennsylvania Department of Highways further specifies that the Bureau of Public Roads formula should be used for beams not covered by the standard.

The British First Report on Prestressed Concrete recommends a total shrinkage strain of 0.0003 in/in and a total creep strain of 0.0004 in/in/ksi of stress in concrete for pre-tensioned members.¹⁴

Many factors such as strength of concrete, water to cement ratio, shape of the beam, and magnitude and distribution of applied stress affect the final loss in prestress. Most of the current methods used to evaluate losses represent only an average condition, and do not respond to specific variation of these factors. In an effort to establish a more rational basis for the estimation of prestress losses, a research project is being conducted in the Fritz Engineering Laboratory at Lehigh University under the joint sponsorship of the Pennsylvania Department of Highways and the U.S. Bureau of Public Roads. The main emphasis is placed on pre-tensioned concrete bridge beams cast in Pennsylvania.

1.2 Object

As a preliminary part, the concretes produced at all of the prestressed concrete producers in Pennsylvania were compared for their potential in contributing to prestress losses. The primary object of the pilot test was to select the two manufacturers of prestressed concrete in Pennsylvania whose products,

because of the quality of the ingredients and their proportions, exhibited the least and the most amount of shortening due to elastic shortening, creep, and shrinkage. The specimens for the main part of the project will then be fabricated at these two plants. Therefore, the final results of the main tests should establish the upper and the lower bounds for the losses in pre-tensioned concrete beams cast in Pennsylvania.

1.3 Scope

The pilot test was conducted on small specimens of prestressed concrete placed at each of the seven manufacturers of prestressed concrete in Pennsylvania. These manufacturers are listed alphabetically in Table 1. Each series of the specimens as cast was designated by a letter from B to T. All details reported herein will be referred to the letter designation rather than the name of the producers.

Besides the composition of concrete, the following factors also influence the creep and shrinkage in concrete:

1. Magnitude and distribution of applied stress
2. Time when stress is first applied
3. Shape and size of specimen
4. Storage humidity
5. Storage temperature

In the preliminary study, all of these factors were kept constant.

Since the pilot test was to compare different concrete products, a strict schedule was followed to make sure that all of the specimens were treated the same way. The relative magnitudes of creep and shrinkage measurements rather than their absolute values were of primary concern.

Before describing the pilot study, a review of the fundamental theories related to creep and shrinkage in concrete and a summary of some of the previous research projects in related subjects will be in order.

2. CREEP AND SHRINKAGE

2.1 Introduction

Among the four main factors contributing to the prestress losses in pre-tensioned concrete members, elastic shortening is the best understood. This shortening can be computed and taken care of very easily. Relaxation in steel is not affected by the concrete properties. Therefore, it was not studied in the preliminary part of this project. This chapter is devoted to the phenomena of creep and shrinkage which are the other two contributing factors to the losses of prestress forces. Mechanism of creep and shrinkage, as well as the factors influencing them, are explained. There are many papers published on these two subjects because of their importance in estimation of losses.^{3,7,8,10,13,14,17,18}

2.2 Cement Hydration

Water and cement start to react chemically upon getting into contact with each other. This reaction is known as cement hydration. As a result of hydration, slender crystals form around the cement grain as shown in Fig. 1a. When these crystals from neighboring cement grains touch one another, a bond is formed

which keeps cement grains together as shown in Fig. 1b. The spaces between the crystals are filled with a felted mass of thin, wrinkled foils.¹³ The products of hydration are commonly referred to as cement gel.

Cement gel contains many voids called gel pores. The porosity of cement gel at maturity is estimated to be 28 percent. The diameter of a gel pore is normally between 40 and 50 angstroms, which is only 4 to 5 times the size of a water molecule.⁸ Due to the extreme fineness of the gel pores, a great adhesive force is developed between the gel and the thin layers of water in the pores.

Hydration is a very long-time process. As hydration progresses, the anhydrous cement grains become surrounded by the cement gel of ever-increasing thickness. Water must penetrate the gel before further hydration can take place. Thus, the rate of hydration decreases as time increases.¹³ Complete hydration is almost never achieved.

2.3 Water in Hardened Concrete

The water commonly used in concrete mixtures is about three times that needed for hydration. Before concrete hardens, a portion of this water evaporates while another portion reacts

chemically with the cement during the hydration. The water contained in hardened concrete may be classified into three types according to the manner in which it is attached to the solid particles:

1. crystallization water
2. gel water
3. capillary water¹⁸

Crystallization water is very strongly bound to the solid particles and actually acts as a solid. For all practical purposes, it may be considered irremovable. Gel water is the water held in gel pores. As mentioned previously, it is also held tightly, but it may escape and evaporate under favorable conditions. Capillary water is held in spaces among gel and aggregate particles called capillary cavities. The average size of a capillary cavity is about 100 times larger than a gel pore. Therefore, the absorptive forces exerted by them on capillary water is relatively weak. Hence, water moves into or out of capillary cavities fairly freely.

Water, like any other liquid, tends to move to a low pressure environment. Therefore, when a pressure gradient exists between concrete and its surrounding medium, water tries to move into or out of concrete. The pressure gradient can be created by stresses in the concrete, or by a variation of the relative

humidity which is a measure of vapor pressure. An atmosphere with low relative humidity has a low vapor pressure. Therefore, when a concrete specimen is stored where the relative humidity is less than that of the concrete, water seeps out of the concrete, provided that it can overcome surface tension and frictional forces. As will be discussed in detail later, this seepage of water is one of the main reasons why concrete creeps and shrinks.

2.4 Creep Theories

At the present time, the mechanism of creep is not completely understood. Various theories have been proposed in the last three decades. Several of these theories are briefly discussed here.

Plastic Theory

Plastic theory assumes that the creep of concrete is the result of slipping along planes within the crystal lattice. In other words, creep of concrete is very similar to plastic flow of steel. Bingham and Reiner suggested that mortar acts as an elasto-plastic material with a yield point of 65 psi.¹⁷ Therefore, concrete creeps even at very low stresses.

Volume of any specimen remains constant under plastic flow. However, creep causes a volume change in concrete. Also, plastic flow cannot be recovered while creep of concrete is partially recoverable upon unloading. Therefore, plastic flow cannot count for all of creep.

Viscous Theory

Viscous theory attributed creep to the viscous flow of cement gel under stress. When an external force is applied to a concrete specimen, a fraction of the load is resisted by the aggregates and the rest by cement paste. Under the influence of sustained load, the cement paste moves, and consequently, causes creep.

According to this theory, the viscosity of the cement paste increases as cement hydration continues. Hence, the rate of viscous flow decreases, and so does the rate of creep.

As the viscous flow of cement gel continues, the load is transferred from the gel to the aggregates. Therefore, the stress in aggregate particles increases with time and so does the aggregate strain. Thus, a part of the creep strain in concrete is attributed to the "delayed elastic shortening" of aggregates.¹⁷

Aggregates normally have a much higher modulus of elasticity than the cement paste. As a result, the stress in the aggregates is much more than that in the cement paste. Hence, the aggregates may creep even though the same aggregates do not exhibit any creep under stress equal to the average stress in the concrete specimen.¹⁷ This creep of aggregates also contributes to the total creep of concrete.

Viscous creep is independent of storage humidity while the creep of concrete has been proven to depend on this factor. Therefore, viscous flow is not the only factor causing creep in concrete. Volume change in concrete can also be used to show that viscous flow is not entirely responsible for creep in concrete.

Seepage Theory

According to this theory, creep is the result of the seepage of water from the concrete. As mentioned before, water can escape from both capillary cavities and gel pores.

Removal of water from capillary cavities has little effect on the volume change in concrete. The relatively small surface tension forces would leave these cavities open. Normally, the continued process of cement hydration results in formation of new gel which soon fills the cavities.

Removal of water from gel pores actually causes the concrete member to decrease in volume, but resistance to this movement is very high. Both the adhesive force in the gel pores and the frictional resistance in the water passages must be overcome. As water moves out, the surface tension tends to close the pores, causing the concrete specimen to shrink.

Seepage of water from unloaded concrete causes shrinkage. When concrete is loaded, the rate of water seepage increases, resulting in more shortening. This increased shortening due to the external load is called seepage creep.

According to this theory, the decreasing rate of creep and shrinkage is explained by the following reasons:

1. The amount of water left in concrete decreases with time.
2. Gel pores close up as de-watering occurs.
3. As water moves out, water front gets further away from the surface. Therefore, water has to move a longer distance and to overcome a larger frictional resistance.¹³

Tests have shown that creep occurs even when moisture movement from the concrete to the surrounding medium is prevented. Therefore, seepage creep cannot be wholly responsible for creep.

Viscous-Seepage Theory

As the name implies, this theory assumes that a part of the shortening is due to viscous flow of cement gel while the rest is due to the seepage flow. A. M. Neville suggested that the total creep was a combination of "true" creep (viscous in nature, with a gradual transfer of load to the aggregate) and increased shrinkage (seepage due to evaporation and external force).¹⁷

The ratio of viscous creep to seepage creep varies greatly. The percentage of total creep due to seepage increases as the conditions become more favorable for de-watering. At 100% relative humidity storage, seepage creep reaches its maximum in the first day after loading. As hydration reduces the relative humidity of the concrete, water starts to move back into the concrete. After a period of time, the seepage creep is reduced to zero. At 20% relative humidity, about 50% of creep is due to seepage. This percentage changes with the size and shape of the concrete specimen.

Of the several theories for the creep of concrete discussed here, viscous-seepage theory is the most nearly correct, and is most consistent with test results.

2.5 Factors Influencing Creep and Shrinkage

Creep in concrete has been shown to be greatly influenced by the quality of the ingredients in concrete. An investigation carried out at the University of California showed that creep was the smallest for the concrete made from limestone, followed by quartz, granite, basalt, and sandstone.²¹ The creep of sandstone concrete is more than twice that for limestone concrete. Well graded and well shaped aggregates decrease creep strain. The type of cement used also affects the creep rate. For concretes having the same compressive strength at age of infinity, creep increases in order for concretes made from cement Types III (high early strength), I (normal), and IV (low-heat).

The proportion of the concrete mixture affects the creep rate significantly. As cement paste content increases, so does the creep strain. Creep of concrete also increases directly with the water-to-cement ratio.

As applied stress increases, so does the pressure on cement gel which in turn results in higher viscous and seepage creep. Creep is approximately proportional to the ratio of applied stress to actual strength. Distribution of applied stress also has some bearing on creep strain. As age and maturity of concrete at first loading increases, the creep strain decreases.

As mentioned previously, water must overcome friction in the passage to seep out of concrete. Thus, the lengths of these passageways affect the creep and shrinkage of concrete. The ratio of volume to surface area is a convenient index of the average length of such passages. Slender members have a low ratio of volume to surface area. A recent research carried out at Portland Cement Association Research and Development Laboratory showed that as volume to surface area increases, creep and shrinkage of the concrete members decrease.¹⁰

As storage humidity drops, in other words, as the vapor pressure of the surrounding medium decreases, the conditions become more favorable for water to escape from the concrete. Hence, water moves out of concrete at a faster rate causing more seepage creep and shrinkage. Higher temperature will cause more evaporation of water which in turn increases seepage creep and shrinkage.

2.6 Creep and Shrinkage Formulae

Many formulae have been suggested to relate creep and shrinkage strains and time. Most of these formulae express the strains as a hyperbolic or a logarithmic function of time.

Lorman and Ross suggested that creep and shrinkage at any time t are related to those at infinity by the following hyperbolic equations:

$$\epsilon_s = \frac{\epsilon_{s\infty} t}{N_s + t}$$

$$\epsilon_c = \frac{\epsilon_{c\infty} t}{N_c + t}$$

where $\epsilon_{s\infty}$ and $\epsilon_{c\infty}$ are the final values of shrinkage and creep, and N_s and N_c are the times at which shrinkage and creep strains attain one half of their final values, respectively.¹⁰ According to this formula, $\frac{t}{\epsilon_s}$ and $\frac{t}{\epsilon_c}$ are linearly dependent on t .

Shank proposed the following exponential equation for creep:

$$\epsilon_c = \alpha t^{1/\beta}$$

where α and β are constants which can be evaluated by plotting $\log \epsilon_c$ against $\log t$.³ This plot is a straight line with a slope equal to $1/\beta$. This equation, which assumes an infinite increase in creep, is not accurate after about one year. A similar equation with different coefficients can be written for shrinkage.

The following formula was suggested by McHenry:

$$\epsilon_c = (\alpha + \beta e^{\rho\tau}) (1 - e^{rt})$$

where τ is age at the time of application of load ($\tau > 5$ days), t is time since the application of load, and α , β , ρ , and r are constants that are found by experiment.³

If creep and shrinkage are assumed to be directly proportional to the logarithm of time, the following formulae could be used.²¹

$$\epsilon_s = \alpha \log (t + 1)$$

$$\epsilon_c = \beta \log (t + 1)$$

where α and β are the slopes of shrinkage and creep curves, respectively.

3. PREVIOUS RESEARCH

Many experiments have been conducted to study the creep and shrinkage behavior of concrete, particularly after the wide acceptance of prestressed concrete. Several of these experiments are reviewed in this chapter.

A very comprehensive study of creep and shrinkage of plain concrete was made over a period of 30 years at the University of California by G. E. Troxell, J. M. Raphael, and R. E. Davis, starting in 1926.²¹ Most of their creep specimens were loaded to 800 psi at 28 days. The following is a review of their test variables and results.

1. Aggregate to cement ratio and water to cement ratio:

Aggregate to cement ratio was varied from 4.25 to 6.75 by weight, and water to cement ratio from 0.5 to 0.8 by weight. Shrinkage strain did not seem to be related to either one of these ratios. However, the creep strain increased directly with water to cement ratio and inversely with aggregate to cement ratio.

2. Composition and fineness of cement:

Concretes made from Type IV (low-heat) portland cement showed higher creep and shrinkage than concrete with Type I (normal) portland cement. Shrinkage was also greater for finely ground cement.

3. Mineralogical character of aggregates:

Concrete specimens were fabricated with constant water-cement ratio and aggregate-cement ratio using six different types of aggregates. Shrinkage was the highest for sandstone, followed by gravel, basalt, granite, limestone, and quartz in that order. Creep was also the highest for sandstone, but followed in order by basalt, gravel, granite, quartz, and limestone.

4. Size of aggregates:

Two gradations of gravel aggregates were used. One had a fineness modulus of 5.15 and a maximum size of 3/4 in., and the other a fineness modulus of 5.69 and a maximum size of 1-1/2 inches. A difference was noted in the initial rates of creep, but not the long term creep strains.

5. Size of specimen:

Creep of concrete decreased as the size of cylindrical specimens increased from 6 inches

in diameter and 12 inches long to 10 inches in diameter and 20 inches long.

6. Moisture condition of storage:

Both creep and shrinkage increased significantly as the humidity of the storage decreased.

7. Intensity of loading:

Creep was approximately proportional to stress as the stress increased from 300 psi to 1200 psi. Specific creep (creep per unit stress) was only slightly greater at the high level.

8. Age of concrete at the time of loading:

A significant effect was noted on creep by the age of concrete at loading. After one year under load, the specimens loaded at 90 days showed 20% less creep strain than those loaded at 28 days.

9. Duration of time:

Creep and shrinkage shortenings continued over the entire period of observation with a steadily decreasing rate. On the average, the creep and shrinkage strains were $1/4$, $1/2$, and $3/4$ of those at 20 years after about two weeks, two months, and one year, respectively.

The factors discussed above and many other related factors have been studied separately by others. A series of tests was conducted by Professor Inge Lyse in 1960 on the effect of the amount of cement paste.¹⁵ Testing concrete specimens with cement paste contents of 27 and 33 percent, Lyse concluded that, for design purposes, the shrinkage of concrete may be considered to be directly proportional to the amount of cement paste. A similar conclusion was made for creep strain, provided that the sustained stress is based upon the ultimate strength of the concrete at the time of the application of the load.

U. J. Counto investigated the effect of the elastic modulus of the aggregates on the elastic shortening, creep, and creep recovery of concrete in 1964.⁶ Two series of tests were carried out. The elastic moduli of the coarse aggregates varied from 0.0425×10^6 to 15.2×10^6 psi in the first series, and from 10.5×10^6 to 30×10^6 psi in the second series. The modulus of elasticity of concrete was found to increase with that of the aggregates. Also, considering the concrete to be a mixture of the aggregates and cement mortar, the modulus of elasticity of the mixture increased with the relative volume of the stiffer of the two components. Both the creep strain and the creep recovery were observed to vary inversely with the elastic moduli of the aggregates.

Best and Polivka studied the creep behavior of lightweight concrete in 1959.⁵ A constant stress at 40% of the compressive strength was used. The elastic shortenings of the lightweight concretes were greater than those of the normal weight concretes of comparable strengths. However, the shrinkage and creep of the lightweight concretes were significantly lower. Total strains in lightweight concretes were less than or equal to normal concretes.

P. W. Keene compared the shrinkage of air entrained concrete with that of non-air entrained concrete of similar workability and strength in 1961.¹² The test results indicated that for concretes of medium workability and strength, the shrinkage strain was not affected significantly by air entrainment. The results of this test also showed that moist air curing for one week increased the total shrinkage of concrete.

The influence of size and shape of members on the shrinkage and creep of concrete is being studied at the Portland Cement Association Research and Development Laboratories. T. C. Hansen and A. H. Mattock presented the results of the first four years of this study in 1966.¹⁰ Most of the specimens were either cylindrical or I-shaped. The cylinders had diameters from 4 to 24 inches and lengths from 18 to 58 inches. The depth of the I-shaped members varied from 11.5 to 46 inches while their lengths changed from 63 to 132 inches. The volume

to surface ratio, which was used as a measure of the relative thickness of the specimens, lay within the range of one to six inches. The creep specimens were loaded to about 25% of their ultimate strengths. The results showed that when de-watering was permitted, the creep and shrinkage of concrete were influenced greatly by the volume to surface ratio. However, the influence of this ratio on the rate of creep appeared to be restricted to an initial period of about three months only. After this initial period, the rate of creep of all members was equal to that of a sealed specimen which had an effective volume to surface ratio of infinity.

J. R. Keeton made an extensive study of the effects of the size of specimens and the relative humidity of storage on creep and shrinkage of concrete, and reported his findings in 1965.¹³ Five different controlled relative humidity environments were used: 20% RH, 50% RH, 75% RH, 97% RH, and 100% RH. The applied stresses were 500, 1000, 2000, and 3000 psi while the design strength was 5000 psi after 7 days in 100% curing. The specimens were cylinders of three sizes: 3 inches in diameter by 9 inches long, 4 inches by 12 inches, and 6 inches by 18 inches. His findings on the size of specimens were essentially the same as those by Hansen and Mattock. Relative humidity of storage environment was observed to have an inverse

but non-linear influence on the volumetric change in concrete. The results of similar tests on hollow box and other shapes showed the same results.

The storage temperature, as well as its humidity, influences the creep and shrinkage in concrete. K. W. Nasser and A. M. Neville investigated the creep of concrete at elevated temperatures in 1965.¹⁶ Seven temperatures ranging from 70° to 205° F. were used. The results showed that the temperature of storage has a definite effect on the creep of concrete. The rate of creep increased with the temperature up to about 160° F. when it was about 3.5 times that at 70° Fahrenheit. Above 160° F. the creep rate decreased with increase in temperature.

S. Arthanari and C. W. Yu studied the creep of concrete under uniaxial as well as biaxial stresses at high temperatures in 1967.⁴ The temperature range used here was also from about 70° F. to 200° Fahrenheit. For both uniaxially and biaxially loaded specimens, the specific creep varied almost linearly with temperature throughout this range. They also found the final creep strain to be higher with the temperature increasing in finite steps as compared to that due to a constant temperature maintained at the maximum.

T. C. Hansen studied the effect of wind on creep and shrinkage of hardened cement mortar and concrete in 1966.⁹ Little difference was observed in weight change, shrinkage, and creep of cement mortar specimens, whether exposed to wind of 5 m/sec speed or stored in calm air.

Many experiments have been conducted on the influence of the magnitude of the applied stress on creep. One of the most extensive studies was done by J. R. Keeton in 1965.¹³ Relationship between the applied stress and the creep strain was found to be non-linear for all storage humidities except in 100% RH where linearity was reached beginning at 175 days. However, the curvature of the creep versus applied stress diagram, for all storage conditions, was very small and could be neglected for all practical purposes. I. Lyse also found creep strain to be directly proportional to the applied stress.¹⁵

Creep of concrete under varying stress is of special interest in prestressed concrete design. Professor A. D. Ross conducted a test to study this phenomenon in 1958.²⁰ One series of specimens was subjected to constant stresses at various ages. Test results showed that creep strain was approximately proportional to stress, and the total creep strain decreased with an increase in the age of concrete at the time of the application

of stress. In a second series, stress was altered at various time intervals. The residual strain that remained after the removal of all loads was greater for a declining stress than for an increasing stress. Ross used three methods to compute creep under varying stress from creep data under constant stress, and compared the results with the experimental values. Two of his methods gave results rather close to the test data and will be explained here.

In the rate of creep method, the following formula was used to estimate creep at time t after the application of loading:

$$\epsilon_c = \int_0^t f \frac{d\epsilon_1}{dT} dT$$

where $\frac{d\epsilon_1}{dT}$ was the rate of specific creep (creep due to unit stress) and f was the concrete stress. The integration was performed numerically using small time intervals. Total strain at any time was the sum of the elastic strain plus the cumulative creep up to that time. This method disregards stress history of concrete. It also predicts no creep recovery because

$$f \frac{d\epsilon_1}{dT} = 0$$

when $f = 0$. Modulus of elasticity of concrete is assumed to be constant and creep rate is taken to be directly proportional to stress level. This method over-estimates the creep strain under a declining stress and underestimates under an increasing stress. In either case, the results obtained by this method are reasonably close to the test results.

The method of superposition may be explained by an example. Suppose a specimen is stressed to 2000 psi at the age of 28 days, and after 32 days under the constant stress, the stress is reduced to 1500 psi at the age of 60 days. The elastic and creep strain of this specimen is computed by algebraically adding the strain due to a 2000 psi compressive stress applied at 28 days and that due to a 500 psi tensile stress applied at the age of 60 days. This method assumes that specific creep is the same in tension and compression. Also the creep behavior of concrete is assumed to be independent of the stress history. The results obtained by this method agree very closely with test data. Ross concluded that the method of superposition was superior to the rate of creep method. However, its application requires a great amount of experimental data and tedious calculations.

Professor A. D. Ross also investigated the creep of concrete under biaxial stresses in 1954.¹⁹ Specimens under

biaxial compression as well as those under stresses of opposite senses were included. Control specimens were subjected to uniaxial compression and tension. The results showed that creep in one direction was not affected by stress in the other direction. Also specific creep for uniaxial tension and compression were found to be the same, substantiating the basic assumption of the method of superposition described in the foregoing paragraph.

J. M. Illston studied the creep of concrete under uniaxial tension more extensively in 1965.¹¹ The effects of stress level, period under load, age of concrete at loading, and the relative humidity of the environment were included. Creep was found to be proportional to tensile stress up to approximately 50% of the ultimate tensile strength. The initial rate of tensile creep was higher than that of compressive creep. But the rate of tensile creep decreased much more rapidly. After a period of time, it became less than the rate of compressive creep. The total tensile creep strain was found to be higher than the total compressive creep strain. The rate of creep for concretes loaded at different ages were virtually identical. Also, the effect of relative humidity of the storage was found to be negligible.

4. DESCRIPTION OF EXPERIMENT

4.1 First Phase

4.1.1 Purpose

In order to compare the creep and shrinkage characteristics, concrete specimens were cast at each of the seven manufacturers of prestressed concrete in Pennsylvania. All creep specimens were post-tensioned to an initial stress of 2000 psi while shrinkage specimens were free of loading. Creep and shrinkage strains were measured and the two manufacturers whose specimens exhibited the largest and the smallest amount of shortening were identified.

For most of the standard bridge members of the Pennsylvania Department of Highways, the average prestress is between 1000 and 2000 psi. The stress level of 2000 psi was chosen near the upper limit of this range in order to facilitate the comparison.

4.1.2 Test Specimens

The creep and shrinkage specimens were 8-in. diameter cylinders 24 inches long with a 1.5-in. concentric hole for the

post-tensioning bar, as shown in Fig. 2. One series of specimens were cast at each one of the seven plants. Each series contained eight specimens, four from one batch of concrete, and the rest from another batch. One specimen from each batch was kept unloaded for shrinkage measurements. The remaining three were post-tensioned for creep measurements.

Ease of handling was one of the factors considered in choosing the cross-section of the specimens. As the relative rather than the absolute strains were the main concern in the comparison, the actual shape and size of the specimens was of little significance provided that they were the same for all specimens.

4.1.3 Materials

Molds for the cylindrical specimens were fiber tubes manufactured by Sonoco Products Company. They were lined with aluminum foil to prevent moisture from escaping. Steel base plates 12 inches by 12 inches were connected to the molds by four small angles. The steel plates and the inside of the tubes were thoroughly oiled and waxed to prevent bonding with concrete. A 1-1/2-in. aluminum flexible duct was used to form the center hole. Aluminum hollow cylinders with 1-3/8-in. outside diameter

were used inside the ducts to keep them straight during the placement of concrete. These hollow cylinders were held in place at both ends. At the bottom they were fastened to the bearing plates. At the top they were attached to 2-in. by 12-in. steel plates which were in turn fastened to the top of the side forms by bolted angles. Figure 3 shows a view of the concrete forms.

The concrete mixtures were the same as those used by the plants to fabricate bridge beams for the Pennsylvania Department of Highways. Relevant information on proportions and properties of the concrete ingredients and mixtures are shown in Tables 2 and 3. The specimens were cured in the plants using the same procedure as was used for highway bridge beams. The curing period was normally about 20 hours at 140° F. temperature.

Post-tensioning was achieved by means of 1-3/8-in. high strength steel bars manufactured by Stressteel Corporation. Some mechanical properties of the steel used are listed below:

ultimate strength, f'_s	158.6 ksi
proportional limit	105.2 ksi
modulus of elasticity	32,300 ksi

Tests done on Stressteel bars by the manufacturer indicates that relaxation at 0.7 ultimate strength does not exceed 2 percent.

As the initial stress on the bars in this study was only $0.41 f'_s$, the relaxation loss was considered negligible.

4.1.4 Instrumentation

The creep and shrinkage shortenings of specimens were measured by a Whittemore strain indicator over the middle 10 inches. Four sets of target points were installed at 90° angles as shown in Fig. 2. Each target point consisted of a brass insert and a stainless steel contact seat. The brass inserts were coated with a layer of fine sand to improve bond with concrete. They were then attached to the inside of the cylinder molds before the placing of concrete. After the forms were stripped, the contact seats were screwed to the inserts. Figure 4 shows a brass insert, before and after coating, and a contact seat. To maintain the small tolerance in the 10-in. gage lengths required for the Whittemore strain indicator, steel spacer bars were used for the attachment of the inserts.

4.1.5 Specimen Preparation

After curing for approximately 20 hours, the specimens were transported to Fritz Engineering Laboratory. They were stripped of forms and their ends were ground smooth. Then the contact seats were placed and the initial gage readings were

taken. After this, the three creep specimens from each batch were laid end to end on a horizontal I-beam, with approximately 1/4-in. gap between specimens. The prestressing bar was then passed through the center hole of all six specimens. The bar was lubricated thoroughly to keep the friction force to a minimum. The creep specimens were then grouted together by hydrostone. A time interval of about 3 hours was allowed for the hydrostone paste to harden. The creep specimens were thus formed into a 12-ft. column.

4.1.6 Post-tensioning

Immediately before post-tensioning, the 12-ft. column was lifted a few inches to release any restraining effect of the I-beam on the shrinkage of specimens and a set of strain readings was taken. The setup for post-tensioning, which was done at approximately 48 hours after casting, is shown in Fig. 5. The load cell consisted of a 1-3/8-in. stressteel bar with two pairs of foil strain gages. The load was applied by a hydraulic jack at the end opposite that where the load cell was placed. When the load cell showed the desired force in the bar, the nut on the jacking end was tightened and the jack was released. The column was again lifted a few inches to allow the specimens to adjust to their new lengths after elastic shortening. Whittemore gage readings were immediately taken. The column was then

placed in a vertical position, as shown in Fig. 6. Strain readings were then taken at regular intervals.

The load cells were under load continuously so that the variation of prestressing force with time could be recorded. However, the zero readings of the load cells changed after a period of time. Therefore, while the load cell readings at post-tensioning time were accurate, those after a period of time were not dependable.

4.2 Second Phase

4.2.1 Purpose and Scope

In the first phase of the study, Series B, C, and D all showed rather high creep and shrinkage strains, but neither of the three shortened significantly more than the other two. (See Chapter 5 for discussion of results.) Therefore, it was decided to extend this preliminary study.

The second phase of the pilot test included concrete specimens from manufacturers B, C, D, and E. Plant E was included because its concrete exhibited the lowest volumetric change in the first phase. This new phase of the project contained three parts.

1. Specimens cast at Plants B, C, D, and E were repeated to check the consistency of the results.
2. It is known that creep rate decreases with the ratio of the applied stress to the actual strength. Since concretes from different manufacturers showed significantly different strength properties, it was decided to explore this effect. Whenever a concrete exhibited a release strength exceeding the design value by more than 15%, an additional series of specimens was cast and post-tensioned to 40% of actual concrete strength. (The previously mentioned stress value of 2000 psi corresponds to 40% of the design concrete strength of 5000 psi.)
3. Originally, it was planned that the aggregates which exhibit the highest volumetric change would be transported to one designated plant for the fabrication of the main specimens. In order to examine the effects of such transportation of materials, specimens were also produced at Plant E, using the materials transported from Plants B, C, and D.

4.2.2 Specimens and Procedures

The shape and size of the specimens, the materials used, and the instrumentation were all identical to those of

the first phase. For the third part of the extended study, where transported materials were involved, the mixture designs of the originating plants were used. The materials were measured in the plants and transported to Plant E in separate compartments of Gen-way bags which were specifically designed for this purpose. Admixtures were also measured and sent in separate containers. The aggregates, cement, water, and the admixtures were mixed at Plant E in a truck mixer. These specimens were steam-cured alongside the beams cast on the same day at Plant E. The specimens were brought to Fritz Laboratory after about 22 hours of curing. Preparation at Fritz Engineering Laboratory was the same as in the first phase. Tables 2 and 4 give the relevant information on proportions and properties of the concrete ingredients and mixtures used in this phase.

4.2.3 Post-tensioning

Load cells were not used in the second phase as their usefulness in the first phase was rather doubtful. The post-tensioning process for this phase was also slightly different from the first phase. Figure 7 shows the post-tensioning set-up for the second phase. The prestressing force was measured at both ends of the column during post-tensioning. Hydraulic jacks were also used at both ends of the column. The specimens were jacked at one end until the load cell at the opposite end showed the desired force. The nut at the measuring end would

then be tightened. Upon the release of the jack at the dead end, the load at the jacking end would fall off due to the anchorage loss. The jacking force was then increased to the desired value before the nut at this end was tightened. This procedure insured a more uniform force in the post-tensioning bar.

5. TEST RESULTS AND DISCUSSION

5.1 First Phase

Figure 8 shows the typical time variations of strains of concrete produced at one plant. The two bottom curves show the average of measured shrinkage and total strains (elastic shortening, creep, and shrinkage) of the two batches of concrete made at this plant. The top curve requires further explanation.

The specimens had a rather high percentage of prestressing steel (3.04%). Thus, the initial steel stress was quite low (65.4 ksi which is 45% of specified yield strength). This low level of initial steel stress essentially eliminated any loss due to the relaxation of steel. On the other hand, the percentage loss of prestress due to shrinkage and creep of the concrete was unusually high, up to approximately 30% in five months. In order to compensate for the rapid decrease of prestress force, the top curve in Fig. 8 was constructed to represent the expected total strain if a constant stress were maintained. This was done by estimating the average prestress within any time interval and increasing the creep strain in

that interval by the ratio of the initial stress to the average stress in that time interval. The detailed procedure for this adjustment is given in the Appendix.

Shrinkage strains of the concrete specimens fabricated at different plants are shown in Fig. 9, while the total strains of these specimens adjusted to a constant prestress are compared in Fig. 10. Some of the shrinkage specimens showed an expansion during the first few days. This is believed to be a temperature effect. The first readings were taken as soon as the forms were stripped, shortly after the specimens arrived at the Fritz Laboratory. It is probable that thermal equilibrium had not yet been achieved at that time and some thermal expansion was developed afterwards.

Specimens from Series B showed far more shrinkage than any other series. However, Series D was slightly higher in the adjusted total strains. Comparison of these results showed that Series D had the highest creep. Series C also had high creep and shrinkage strains. The total strains in Series B, C, and D were very close and no definite conclusion could be drawn on the one concrete which possessed the highest potential for prestress losses. The second phase of this pilot study was conducted to resolve this problem.

5.2 Second Phase

The solid line, Curve A, in Fig. 11 shows the typical time variation of total measured strain of concrete produced in the second phase. The total strain decreased significantly during the first week before the expected gradual increase took place. It is believed that the thermal expansion, as explained in the previous section, is responsible for a part of this decrease. Also excessive creep of the hydrostone joints between the concrete specimens may have contributed to the initial expansion of concrete. The creep of these joints reduced the total length of the specimens assemblage and relieved a portion of the stress in steel. Therefore, the stress in the concrete specimens was reduced, and the specimens recovered some of their elastic strain. Thus, the total strains in the specimens were reduced. In order to correct this effect, Curve A was extended backwards graphically, to intersect the elastic strain line at M, as shown by dotted line in Fig. 11. Then, the entire curve was shifted up a distance MN, as shown by Curve B in Fig. 11. Curve B was used hereafter for comparison. Due to the excessive creep of hydrostone, which was not experimentally determined, the adjustment used for the first phase observations appeared to be an unwarranted refinement and was not made.

Figures 12, 13, 14, and 15 show the total strains as well as the shrinkage strains of all of the specimens fabricated at the plants B, C, D, and E, respectively. The results from the first phase, before correction for constant stress, are also shown in these figures. The results from the two phases in Plants B and E were very consistent. The two curves for Plant B are within one standard deviation of each other. For Plant E, they are essentially coincidental. The results from the specimens cast at Plants C and D in the second phase differed substantially from those made in the first phase. This difference was explained by the fact that substantial changes in quality control were made at these two plants in the meantime.

For Plant E, comparison of the curves for different stress levels revealed the near proportionality between stress and the elastic strain. However, the time rate of total strain did not change appreciably with the increase in the concrete stress. For Plant D, both the rate of change in the total strain and the elastic shortening increased with the level of stress.

Concrete specimens made from transported material from Plants B and C did not exhibit the same creep and shrinkage characteristics as those placed in the original plants. For Plant C, the specimens with transported material showed higher elastic as

well as total strains. Since the water was added to the aggregates before transportation, approximately one week before mixing, the aggregates had the opportunity to absorb water for a long period of time. A part of this absorbed water might not have been available for workability at the time of mixing. Water was actually added to obtain the desired workability. The resulting high water to cement ratio caused the transported specimens to creep and shrink more. For Plant B, the concrete made from the transported aggregates showed less elastic and total strains. Here water was added to the aggregates only a couple of days before mixing. The concrete mixtures had low consistency (4 to 5 inches slump) and low strength. The characteristics of this concrete were so much different from the concrete cast at Plant B that any comparison would appear meaningless. The specimens with transported aggregates from Plant D showed shortenings very close to those placed at Plant D. In this case, water was not transported, but added at the mixing time. The water content and the slump of the two concretes were about the same.

Figure 16 shows a comparison of the shrinkage of the concrete cast at the four plants, B, C, D, and E. The concrete from Plant B showed far more shrinkage than the concretes placed at the other three plants. Figure 17 shows a comparison of the

total strains of the concretes produced at the four plants. Again, Plant B specimens show more elastic shortening as well as greater rate of change in total strain. Plants C and E showed low elastic shortening and total strain.

The second phase of the pilot test clearly indicated that the concrete placed at Plant B had the highest potential for losses in prestress forces. Hence, it is recommended to use the concrete produced at Plant B to establish the upper bound for prestress losses. Plants C and E both showed the least potential for prestress losses in the second phase. In view of the consistency of results from Plant E between the two phases, it is recommended to use the concrete cast at Plant E to establish the lower bound.

5.3 Comparison of Results and Formulae

The test data from Plants B and E were compared with commonly accepted formulae for creep and shrinkage. The data obtained in the first five days after the placing of concrete were not used because of the effects of thermal equilibrium and the creep of hydrostone discussed previously. In this comparison, both time and strain were referenced to the status five days after casting.

With the reference points so shifted, the data points for the first 100 days for each series of specimens were plotted so that they could be compared with the equations mentioned in Section 2.6. An essentially linear relationship was observed on a log-log scale as shown in Figs. 18 and 19. These lines represent equations of the exponential form $\epsilon = \alpha t^\beta$. Coefficients α and β have been determined graphically for each series, and are shown in the figures.

For both plants, the shrinkage strains for the two phases were nearly the same. Only one straight line was fitted through all of the points for Plant B, while two distinct lines, close to each other, were plotted for the first and second phase of Plant E.

The log-log plot of shrinkage plus creep versus time is shown in Fig. 19. Here the ordinates are the changes in length of creep specimens per unit length; or, the sum of shrinkage and creep due to the varying stress, minus the change in the elastic shortening due to the change in stress. The results of the first and second phase for Plant B plotted into two very close parallel lines. For Plant E, the two lines for the two phases approached each other rapidly. The slope of the shrinkage plus creep curves were higher than the corresponding shrinkage curves with a definite correlation between them.

6. CONCLUSIONS

1. The concrete produced at Plant B has the highest potential for prestress losses.
2. The concrete produced at Plant E has the lowest potential for prestress losses.
3. The concrete produced from transported aggregates does not exhibit the same creep and shrinkage characteristics as those of the originating plant.
4. The plot of the logarithm of creep and shrinkage against the logarithm of time is a straight line for approximately the first 100 days.

7. ACKNOWLEDGMENTS

This study is the preliminary portion of a research project on the prestress losses of pre-tensioned concrete structural members, which is being conducted at the Fritz Engineering Laboratory, Department of Civil Engineering, Lehigh University. Professor D. A. VanHorn, Chairman of the Department, initiated the project in October 1966, and was the Project Director during the first year. Professor L. S. Beedle is the Director of the Laboratory. The Institute of Research, under the directorship of Professor G. R. Jenkins, coordinates all research activities at the University. The sponsors of this research project are the Pennsylvania Department of Highways and the U.S. Bureau of Public Roads.

Acknowledgments are gratefully paid to the seven prestressed concrete plants where the preliminary study specimens were fabricated. Without their wholehearted cooperation, this study would not have been possible.

Thanks are also due to the many members of the Fritz Engineering Laboratory staff who helped in various aspects of this research. In particular, thanks are due to Messrs. S. Cowen,

J. Porcaro, and D. Frederickson for their assistance in the fabrication and observation of the specimens, to Mr. K. Harpel and his staff for their help in handling of the specimens, to Mr. R. Sopko and Miss S. Gubich for photographing and drafting, and to Mrs. Carol Kostenbader for typing this manuscript.

8. NOMENCLATURE

E_s	modulus of elasticity of prestressing steel
f	stress in concrete
f_{cs}	concrete stress at the centroid of the prestressing steel
f_{si}	initial stress in prestressing steel after seating of the anchorage
f'_s	ultimate strength of steel
N_c	time at which the creep strain equals half its final value
N_s	time at which the shrinkage strain equals half its final value
t	time since the application of load
δ_1	ratio of loss in steel stress due to relaxation of prestressing steel
Δf_s	total stress loss in prestressing steel
ϵ_c	strain in concrete due to creep
$\epsilon_{c\infty}$	final stress in concrete due to creep
ϵ_e	strain in concrete due to elastic shortening
ϵ_s	strain in concrete due to shrinkage
$\epsilon_{s\infty}$	final stress in concrete due to shrinkage
τ	age of concrete at the time of the application of load

9. APPENDIX

Adjustment of Creep Strain

The formulae derived in this appendix were used to change the total strain of concrete under varying stress to that due to constant stress.

In deriving the formulae, the following assumptions were made:

1. The modulus of elasticity of concrete (E_c) remains constant.
2. Creep is directly proportional to stress in concrete.
3. At any time, creep strain is uniformly distributed along the entire length of the post-tensioned column.
4. Relaxation of steel is neglected.

The following notation is used in this appendix unless otherwise specified. Strain is represented by ϵ while steel and concrete stresses are shown by f and f' , respectively. The letter subscripts e , s , c , and t used with ϵ indicate, respectively, strain due to elastic shortening, shrinkage, creep, and the total of the three effects. The numerical subscripts 0, 1, 2, and n

indicate the time of observation, with time zero designating the post-tensioning time. Figure 8 shows some of these symbols.

Change in steel stress from post-tensioning time to any time t_1 is

$$\Delta f_{01} = E_s \Delta \epsilon = E_s (\epsilon_{t1} - \epsilon_{t0})$$

Therefore,

$$f_1 = f_0 - \Delta f_{01} = f_0 - E_s (\epsilon_{t1} - \epsilon_{t0}) \quad (1)$$

To find elastic shortening at time t_1 , one can write

$$\frac{\epsilon_{e1}}{\epsilon_{e0}} = \frac{f_1'}{f_0'} \quad (2)$$

but

$$\frac{f_1'}{f_0'} = \frac{(A_s/A_c)f_1}{(A_s/A_c)f_0} = \frac{f_1}{f_0} \quad (3)$$

Substitute equation 3 into equation 2 and rearrange,

$$\epsilon_{e1} = \frac{f_1}{f_0} \epsilon_{e0} \quad (4)$$

Total creep at time t_1 due to varying stress is

$$\epsilon_{c1} = \epsilon_{t1} - \epsilon_{s1} - \epsilon_{e1}$$

where ϵ_{t1} and ϵ_{s1} are directly observed and ϵ_{e1} is computed by equation 4.

Similarly, at time t_2

$$f_2 = f_0 - E_s (\epsilon_{t2} - \epsilon_{t0})$$

$$\epsilon_{t2} = \frac{f_2}{f_0} \epsilon_{e0}$$

$$\epsilon_{c2} = \epsilon_{t2} - \epsilon_{s2} - \epsilon_{e2}$$

The increment of creep during the time interval $t_2 - t_1$ is

$$\Delta \epsilon_{c12} = \epsilon_{c2} - \epsilon_{c1}$$

If the actual stress in steel does not vary greatly during the time interval $t_2 - t_1$, it may be approximated by a constant stress f_{12} which is the average within the interval.

$$f_{12} = \frac{f_1 + f_2}{2}$$

The incremental creep can now be modified to correspond to a constant stress f'_0 .

$$\Delta \epsilon'_{c12} = \Delta \epsilon_{c12} \frac{f'_0}{f_{12}} \quad (5)$$

But

$$\frac{f'_0}{f_{12}} = \frac{(A_s/A_c) f_0}{(A_s/A_c) f_{12}} = \frac{f_0}{f_{12}} = \frac{f_0}{(f_1 + f_2)/2} = \frac{2 f_0}{f_1 + f_2} \quad (6)$$

Substitute equation 6 into equation 5,

$$\Delta \epsilon'_{c12} = \Delta \epsilon_{c12} = \frac{2 f_0}{f_1 + f_2}$$

The total creep strain at any time t_n due to constant stress will be

$$\epsilon'_{cn} = 2 f_0 \sum_{a=1}^n \frac{\epsilon_c(a-1), (a)}{f_{a-1} + f_a}$$

Finally, the total strain due to constant stress at any time t_n is

$$\epsilon'_{tn} = \epsilon'_{cn} + \epsilon_{sn} + \epsilon_{e0}$$

10. TABLES

Table 1

Manufacturers of Prestressed Concrete
in Pennsylvania and Their Locations

1. Dickerson Prestressed Concrete, Inc.	Youngwood
2. Dickerson Prestressed Concrete, Inc.	York
3. Eastern Prestressed Concrete Corporation, Inc.	Hatfield
4. International Pipe and Ceramics Corporation	Erie
5. New Enterprise Stone and Lime Company	Roaring Spring
6. Pottstown Newcrete Products, Inc.	Pottstown
7. Schuylkill Products, Inc.	Cressona

Table 2 Concrete Mixtures

Series	Proportions	Cement Type	Coarse Aggregate	Fine Aggregate	Admixtures		Design Strength (psi)
					Agent	Amount per sack	
B, K, S	1:1.55:2.83	Type III	Crushed Limestone		Plastiment Sika AEA	3.0 oz. 0.75 oz.	5000
C	1:1.39:2.81	Type IIIA	Crushed Limestone	Silica Sand	Pozzoloth APA	0.534 gal. 0.934	(1)
D	1:1.34:3.03	Type III	Crushed Limestone	Crushed Sandstone	Pozzoloth MBVR	0.25 gal. 0.50 oz.	6000
E, L, M	1:1.15:2.41	Type III	Crushed Limestone	Manufactured from Ganister Rock	Plastiment Sika AER	2.0 oz. 1.5 oz.	5500
F	1:1.81:2.73	Type IA	Crushed Limestone	Lake Sand	Pozzoloth	0.25 lb.	(2)
G	1:1.21:2.98	Type IIIA	Crushed Limestone	Silica Sand	Plastiment	3.0 oz.	(1)
H	1:1.18:3.17	Type III	Crushed Limestone	River Sand	Pozzoloth MBVR	0.25 lb. 0.75 oz.	5250
O, P	1:1.40:2.89	Type III	Crushed Limestone	Silica Sand	Pozzoloth MBVR	0.534 gal. 1.5 oz.	(1)
Q, R, T	1:1.40:2.89	Type IIIA	Crushed Limestone	Crushed Sandstone	Pozzoloth MBVR	0.25 gal. 0.53 oz.	5000

Note: First Phase B to H
 Second Phase K to T
 (See page 60 for notes)

Table 3 Concrete Properties (First Phase)

Series	Actual Water Content	Actual Water Content	Slump	Percent Air	Actual Strength	
	gal/cu.yd.	gal/sack			28-day (3)	Release (4)
B	32.9	4.39	1-7/8 2-1/2	4.5 4.2	6670	5200
C	30.2	4.02	1-3/4 2	4.1 4.1	4960	5250 (5)
D	29.9	3.99	2-1/4 2-1/4	3.2 3.3	7900	5558
E	30.0	3.53	1 1-1/4	4.2 4.5	7510	6430
F	32.2	4.60	2-1/8 1-1/2	5.1 4.4	8050	>6000 (6)
G	34.4	4.59	2-1/4 2-1/2	4.8 5.2	6730	6295
H	33.0	4.40	2 2	4.4 5.0	7770	5640

(See next page for notes.)

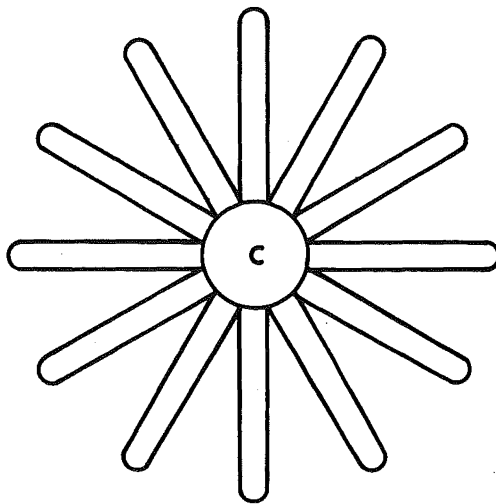
Notes for Tables 2 and 3

- (1) Mix design controlled by specified cement content, 7.5 sacks per cubic yard.
- (2) Mix design controlled by specified cement content, 7 sacks per cubic yard.
- (3) One-day steam curing, 27 days air curing, test conducted by project personnel at Fritz Laboratory.
- (4) One-day steam curing, data supplied by the respective plants.
- (5) The 28-day strength obtained at Fritz Laboratory is lower than the release strength reported by the plant. Strength at ten days (1 day steam, and 9 days air curing) obtained at Fritz Laboratory averaged 4350 psi. Thorough discussion with plant superintendent failed to reveal satisfactory reason for such an abnormal correlation among the strengths.
- (6) Several cylinders did not break at the rated loading capacity of the testing machine.

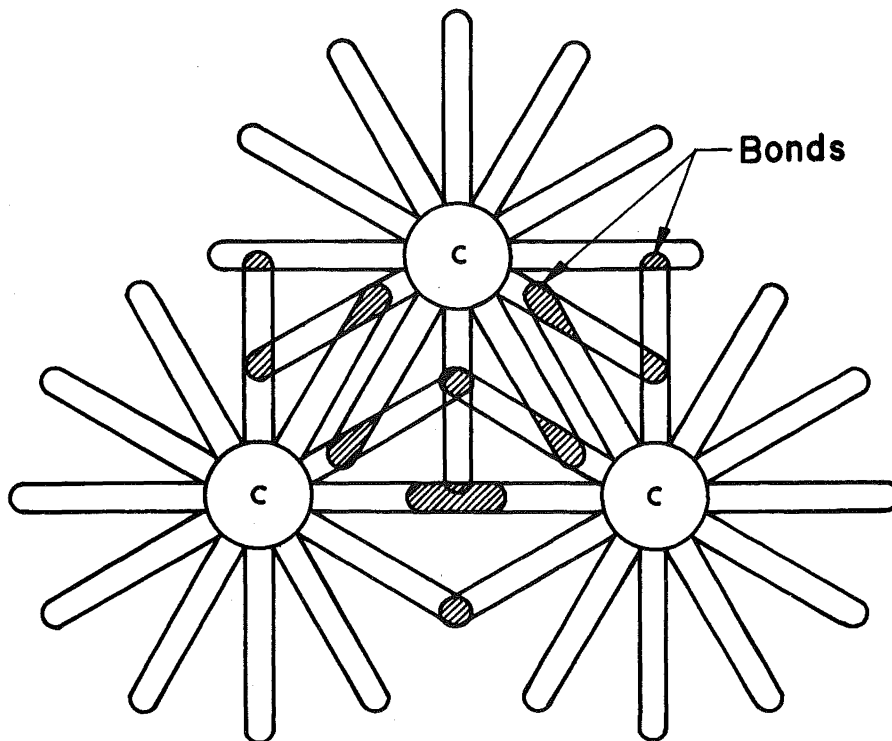
Table 4 Concrete Properties (Second Phase)

Series	Actual Water Content	Actual Water Content	Slump	Percent Air	Actual Strength	
	gal/cu.yd.	gal/sack			28-day	Post-tensioning time
K	a. 29.0	3.87	1-3/4	3.5	6420	5200
	b. 30.0	4.00	2-1/2	4.5	5290	4120
L	a. 29.1	3.88	1-3/4	5.2	7250	6385
	b. 28.2	3.76	1-1/2	4.2	7430	6650
M	a. 29.0	3.87	1-1/4	4.0	7130	6400
	b. 28.7	3.82	1-3/4	3.5	7995	7110
O	a. 27.0	3.61	2	3.0	7460	5960
	b. 27.0	3.61	1	2.9	7430	5910
P	a. 31.6	4.22	1-7/8	2.0	5660	4600
Q	a. 29.2	3.90	2-1/8	3.9	8135	6370
	b. 29.2	3.90	2	3.2	9110	6930
R	a. 29.2	3.90	2-1/2	7.2	5535	4720
	b. 29.2	7.90	1-1/2	4.6	5430	4490
S	a. 32.5	4.35	5	4.8	4990	3890
	b. 31.2	4.17	4	3.5	5780	4350
T	a. 29.3	3.91	2	3.9	8065	6385
	b. 29.7	3.96	2-1/8	3.6	8650	6490

11. FIGURES



a. Initial Growth of Individual Cement Grain



b. Inter-relationship of Growth of Several Cement Grains

Fig. 1 Initial Cement Hydration¹³

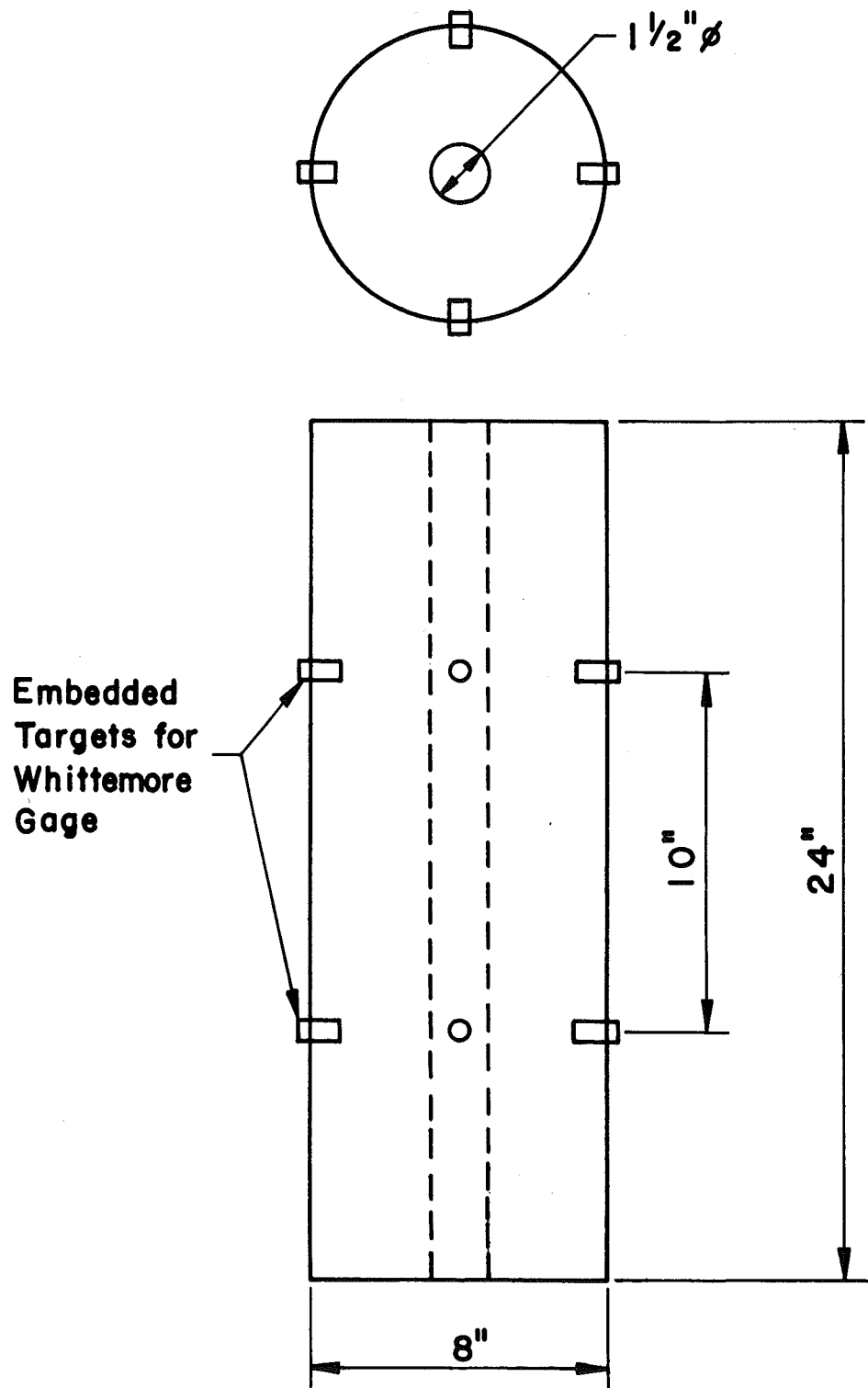


Fig. 2 Creep and Shrinkage Specimen



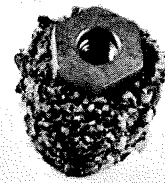
Fig. 3 Concrete Mold



BRASS
INSERT



CONTACT
SEAT



COATED
INSERT

Fig. 4 Brass Insert, Coated Insert, and Contact Seat

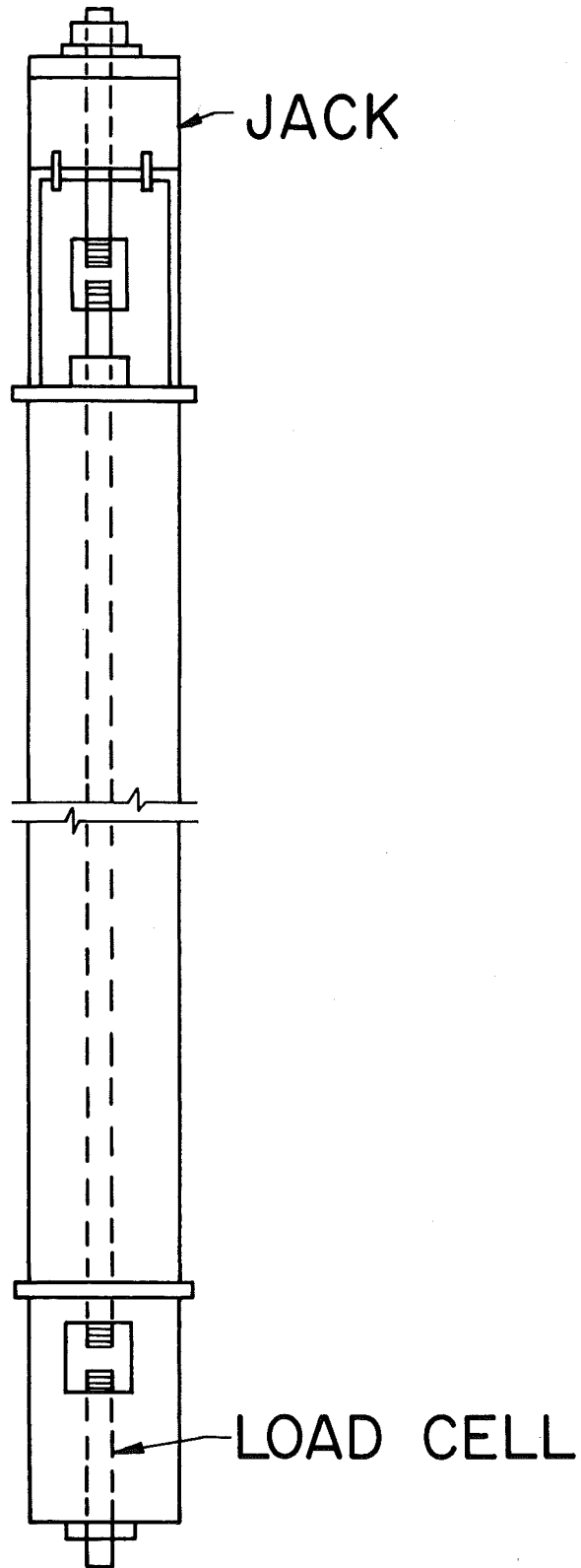


Fig. 5 Post-tensioning Setup for the First Phase

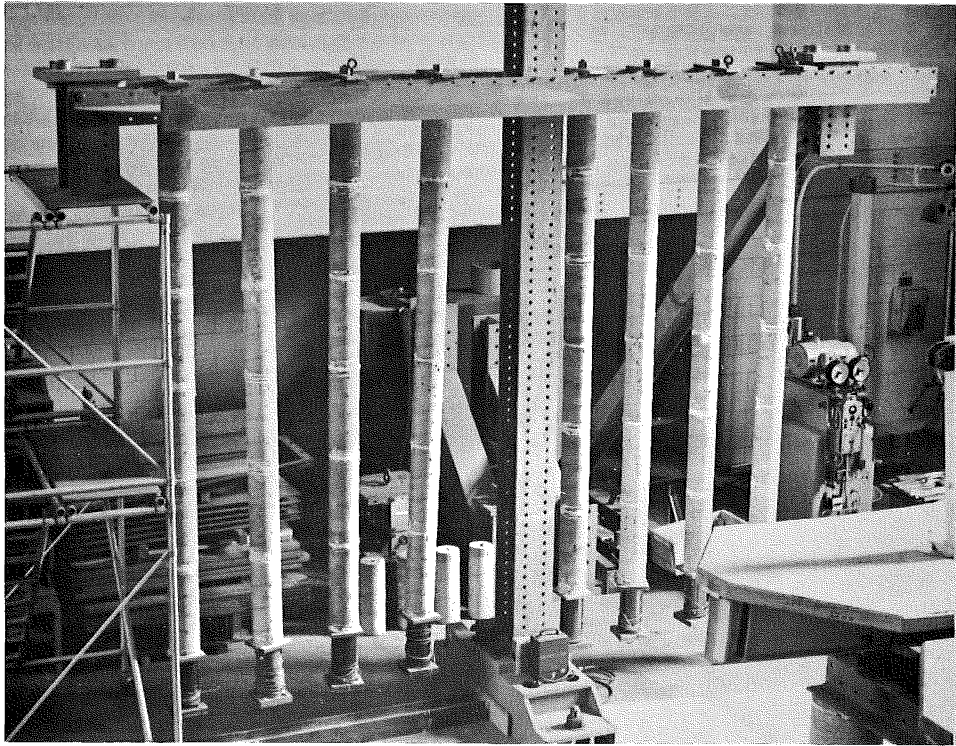


Fig. 6 Specimens After Post-tensioning

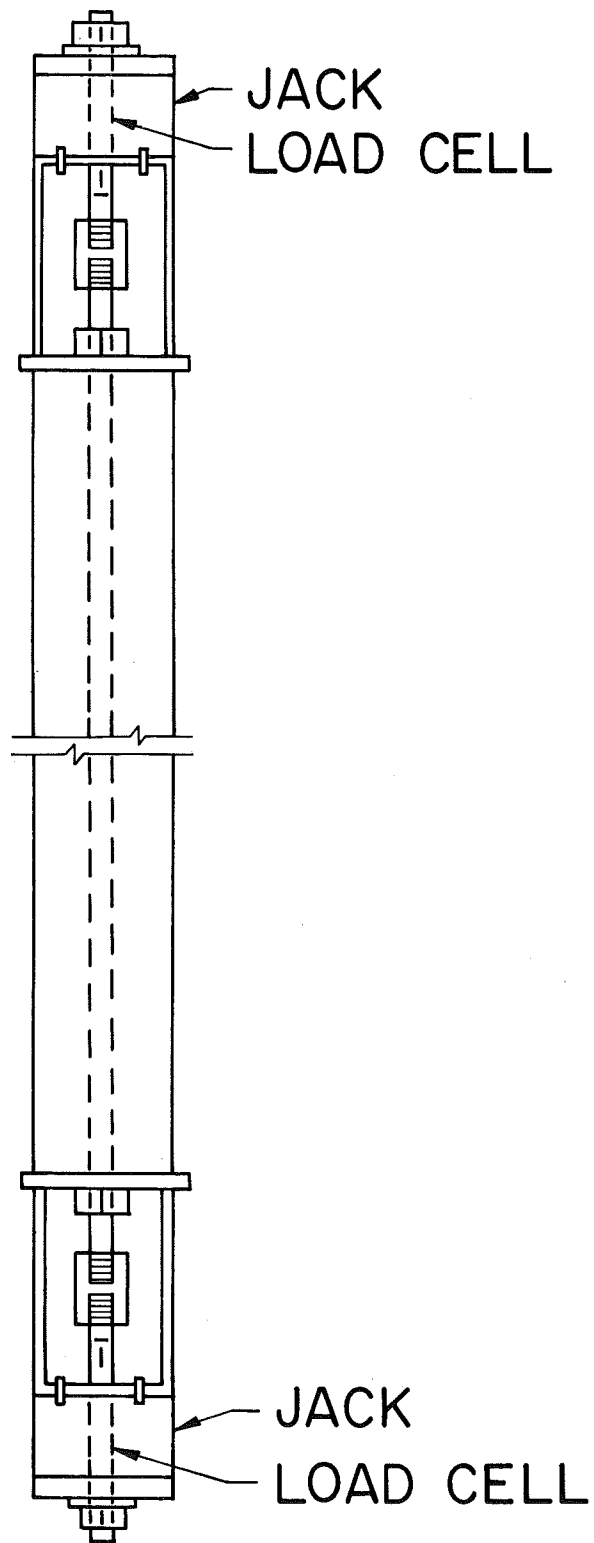


Fig. 7 Post-tensioning Setup for the Second Phase

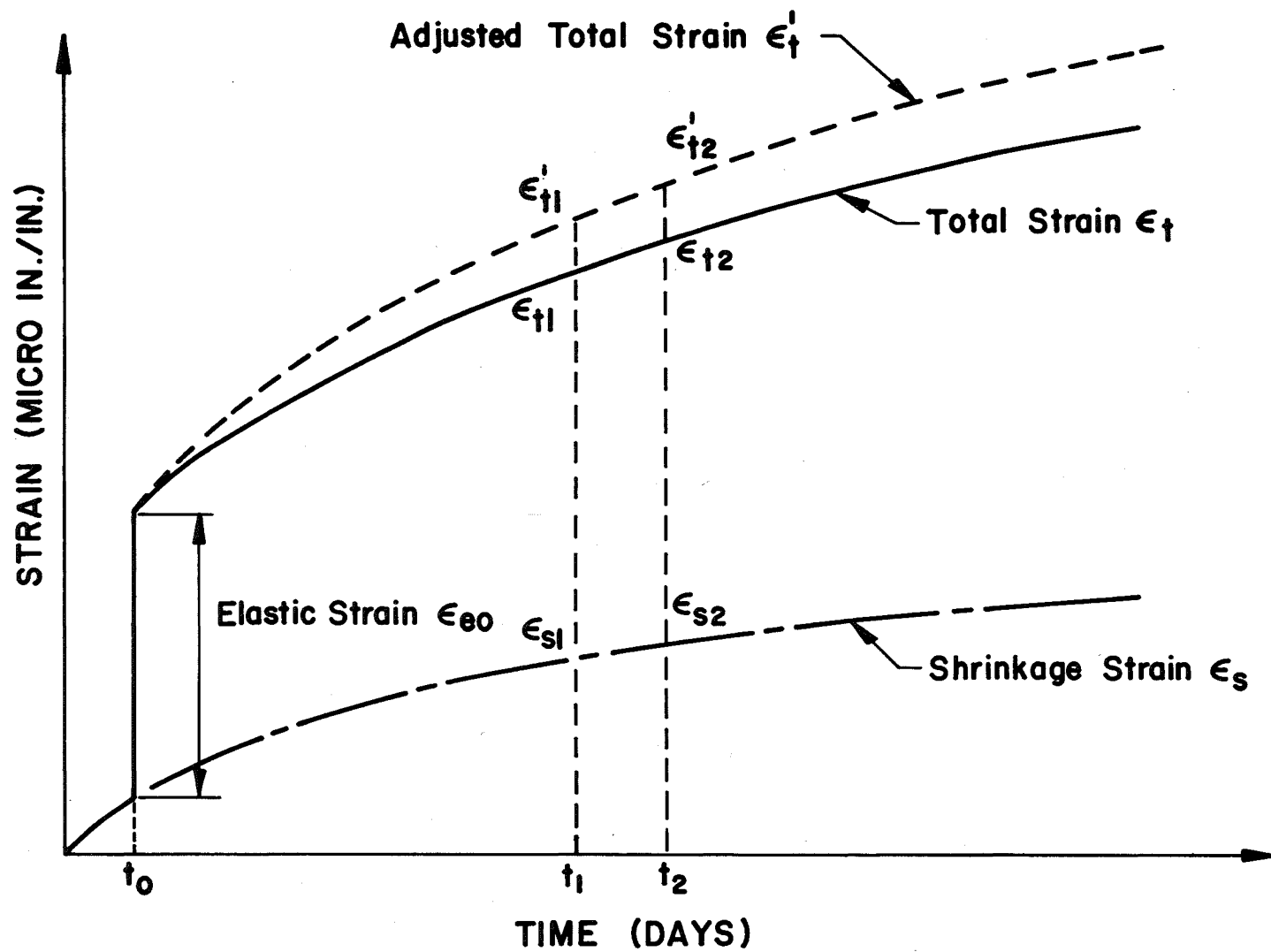


Fig. 8 Typical Time Variation of Shrinkage and Creep Strains

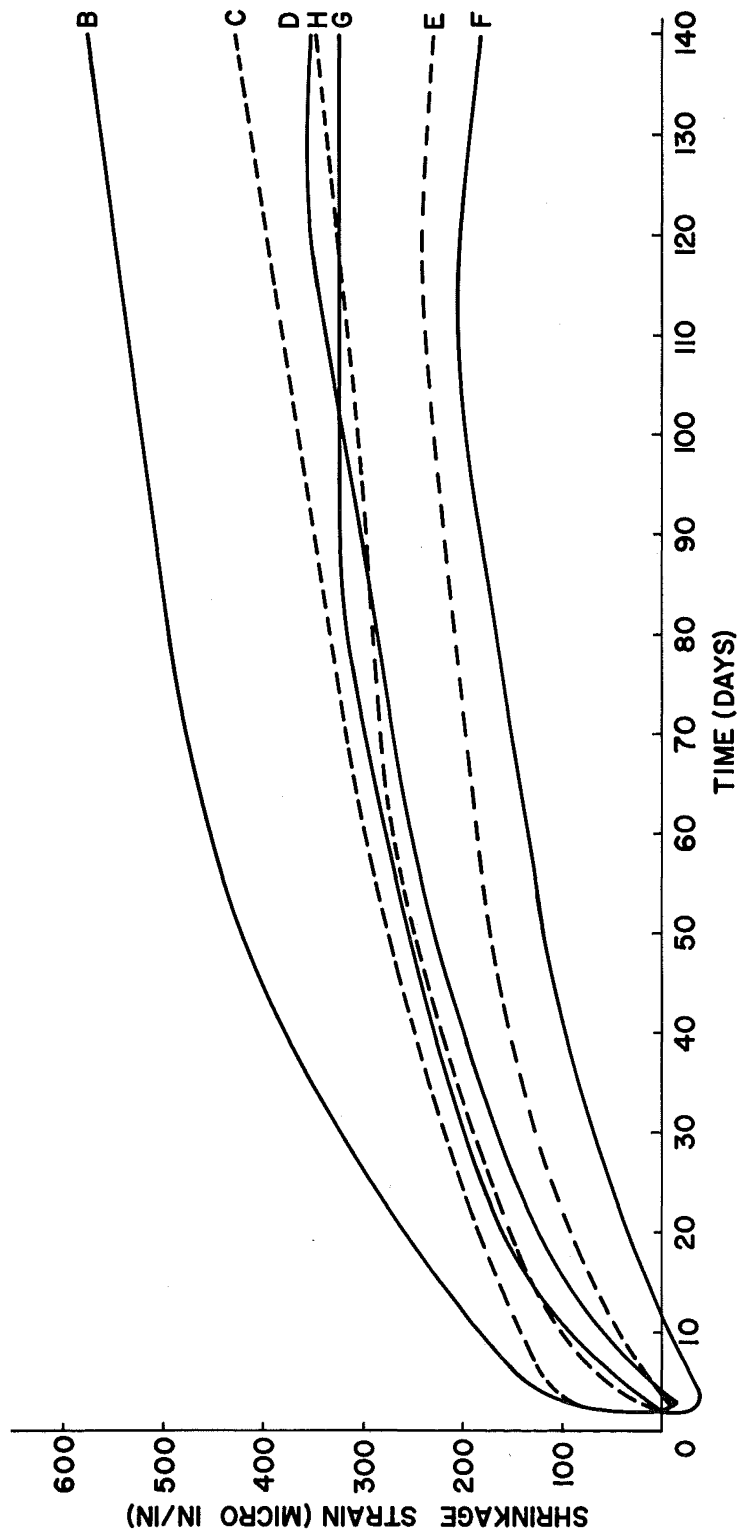


Fig. 9 Shrinkage Strains for the First Phase

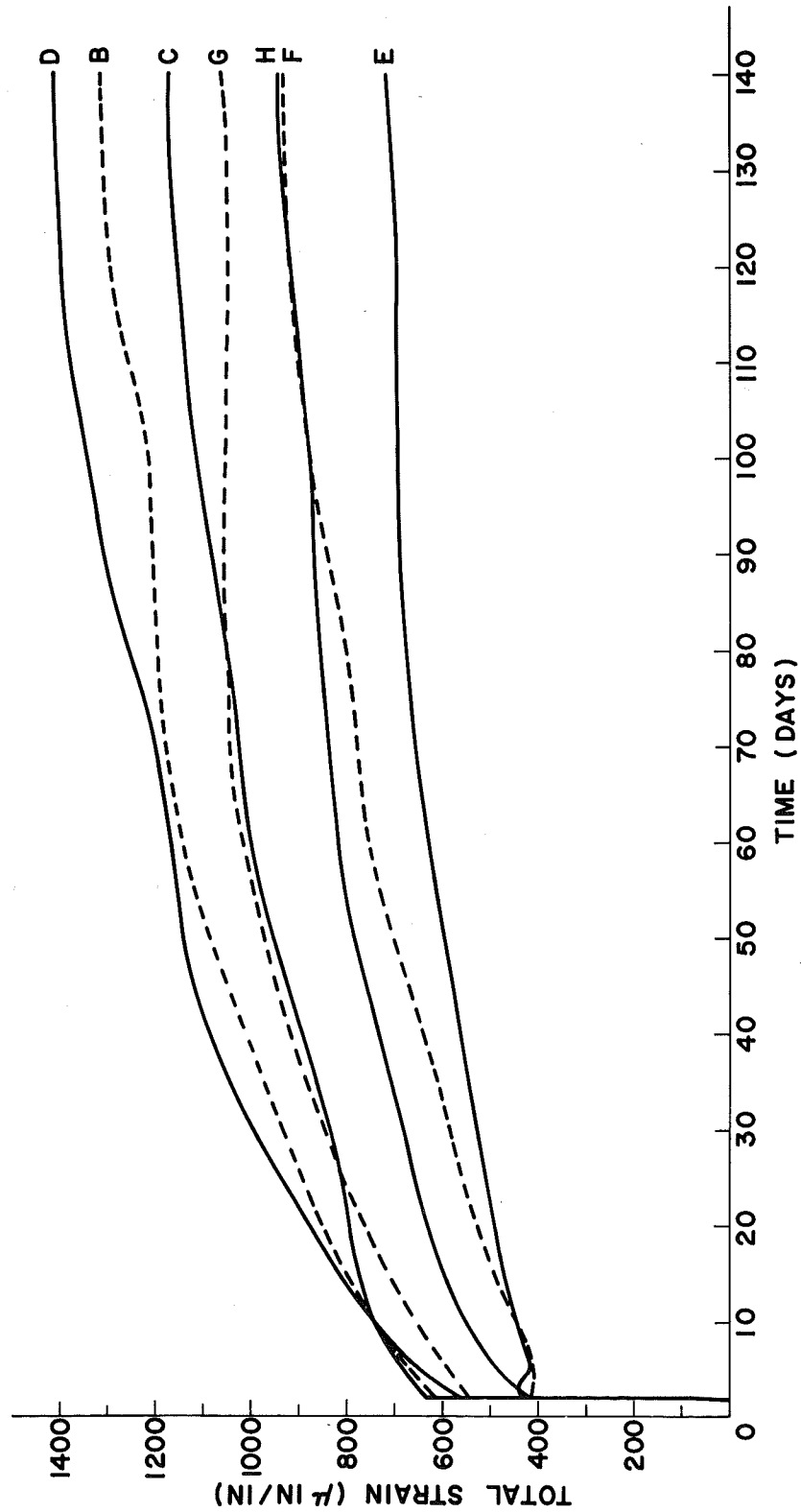


Fig. 10 Adjusted Total Strains for the First Phase

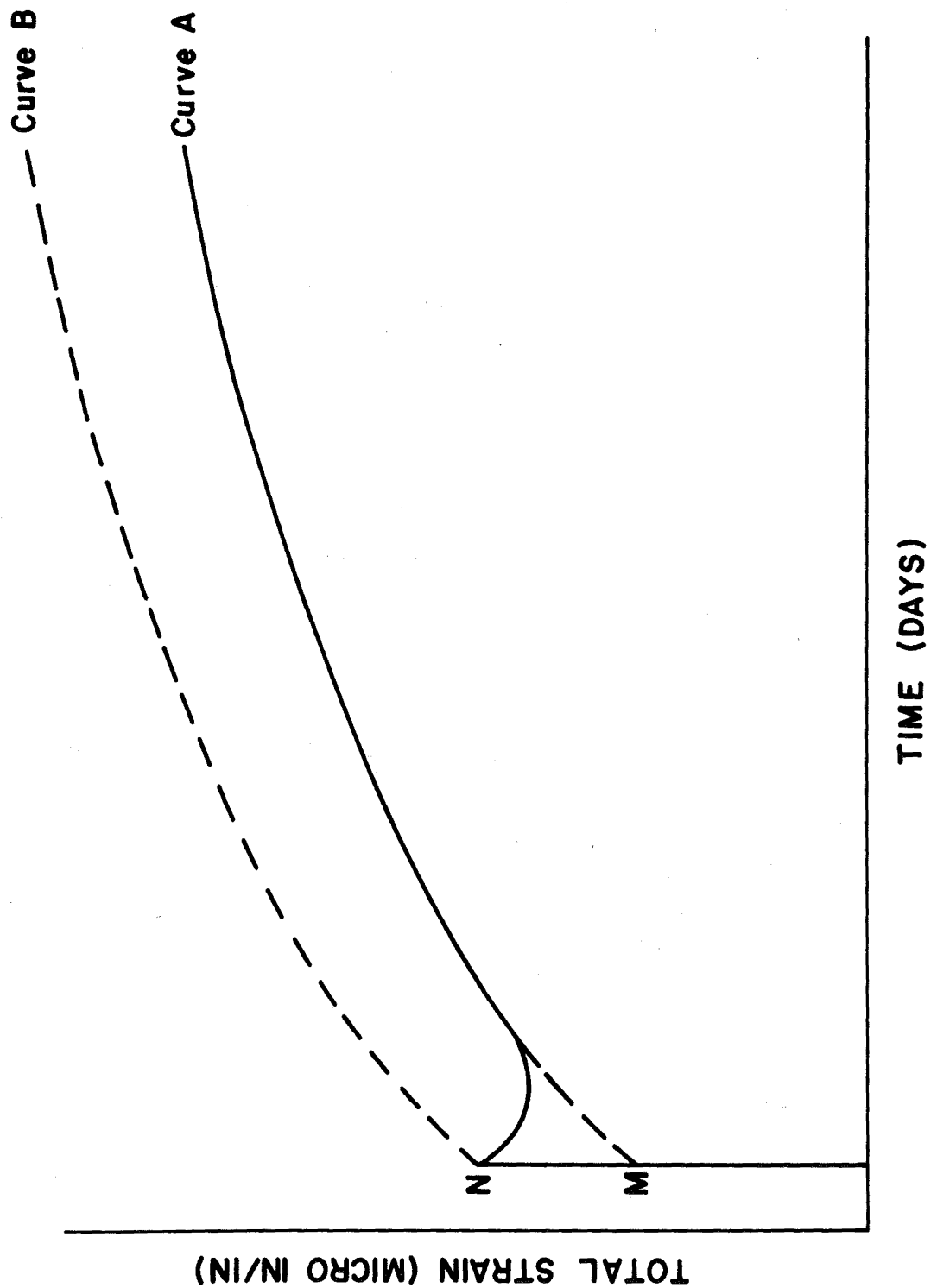


Fig. 11 Correction for the Second Phase Specimens

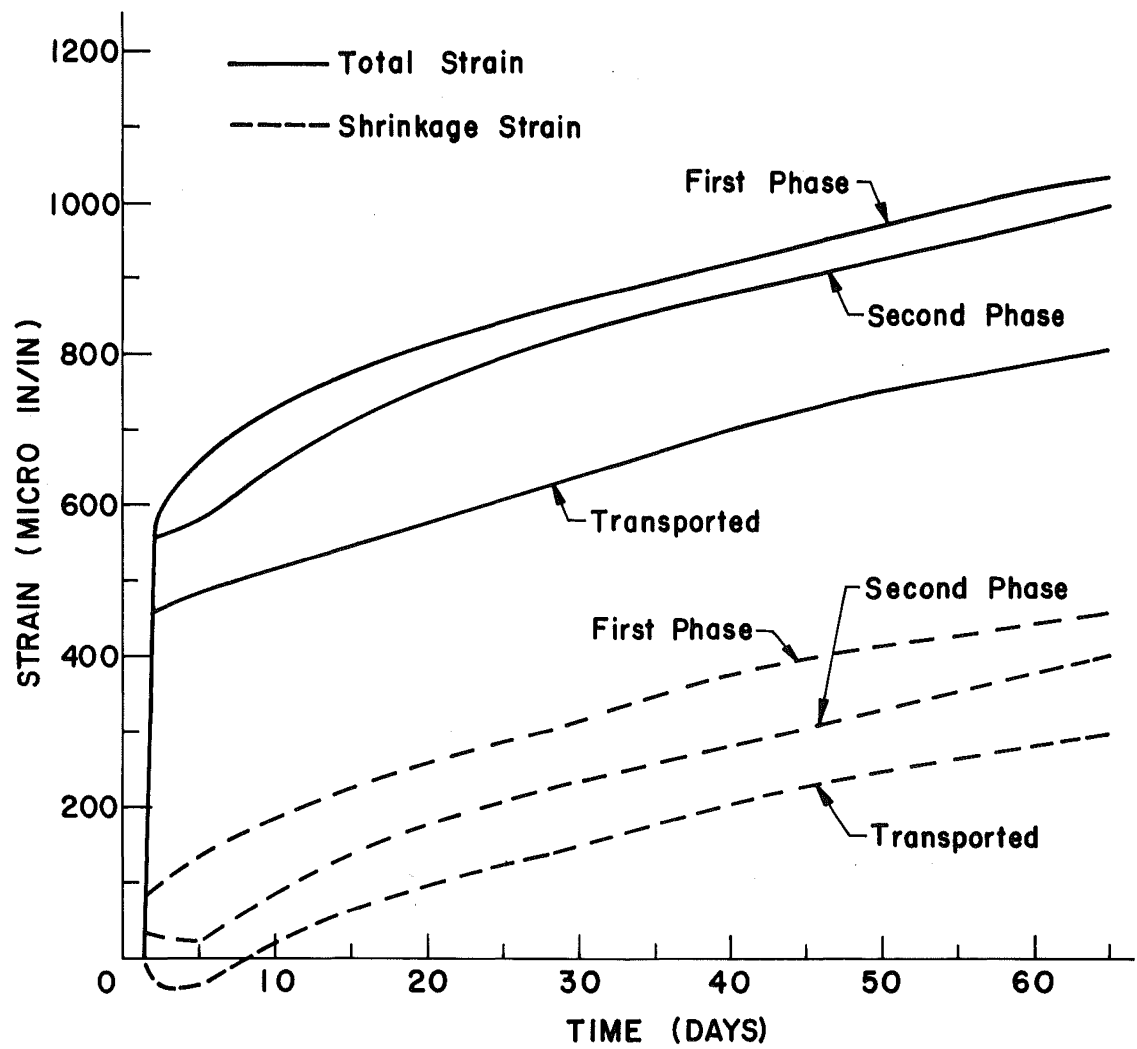


Fig. 12 Total and Shrinkage Strains in Series B

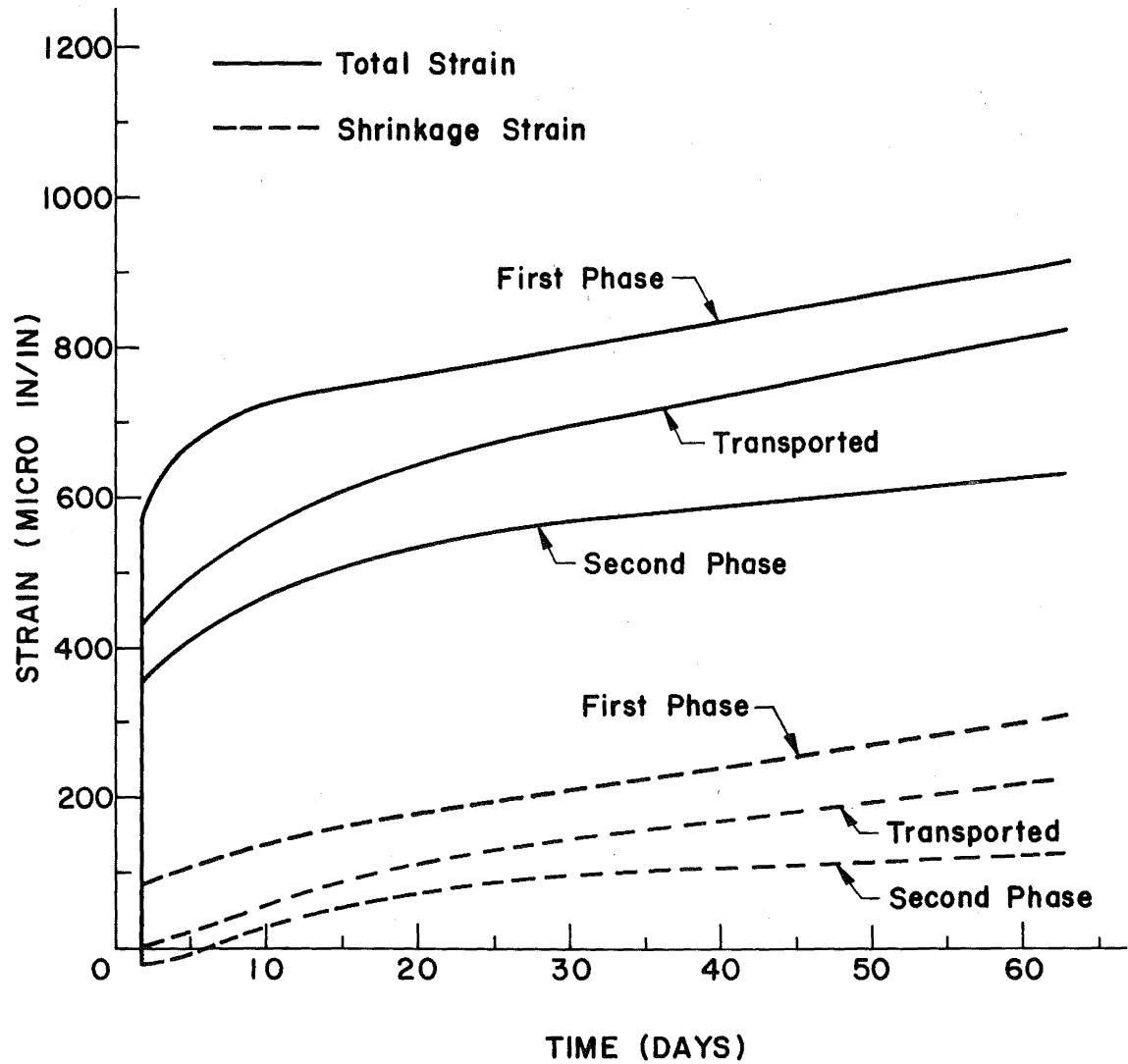


Fig. 13 Total and Shrinkage Strains in Series C

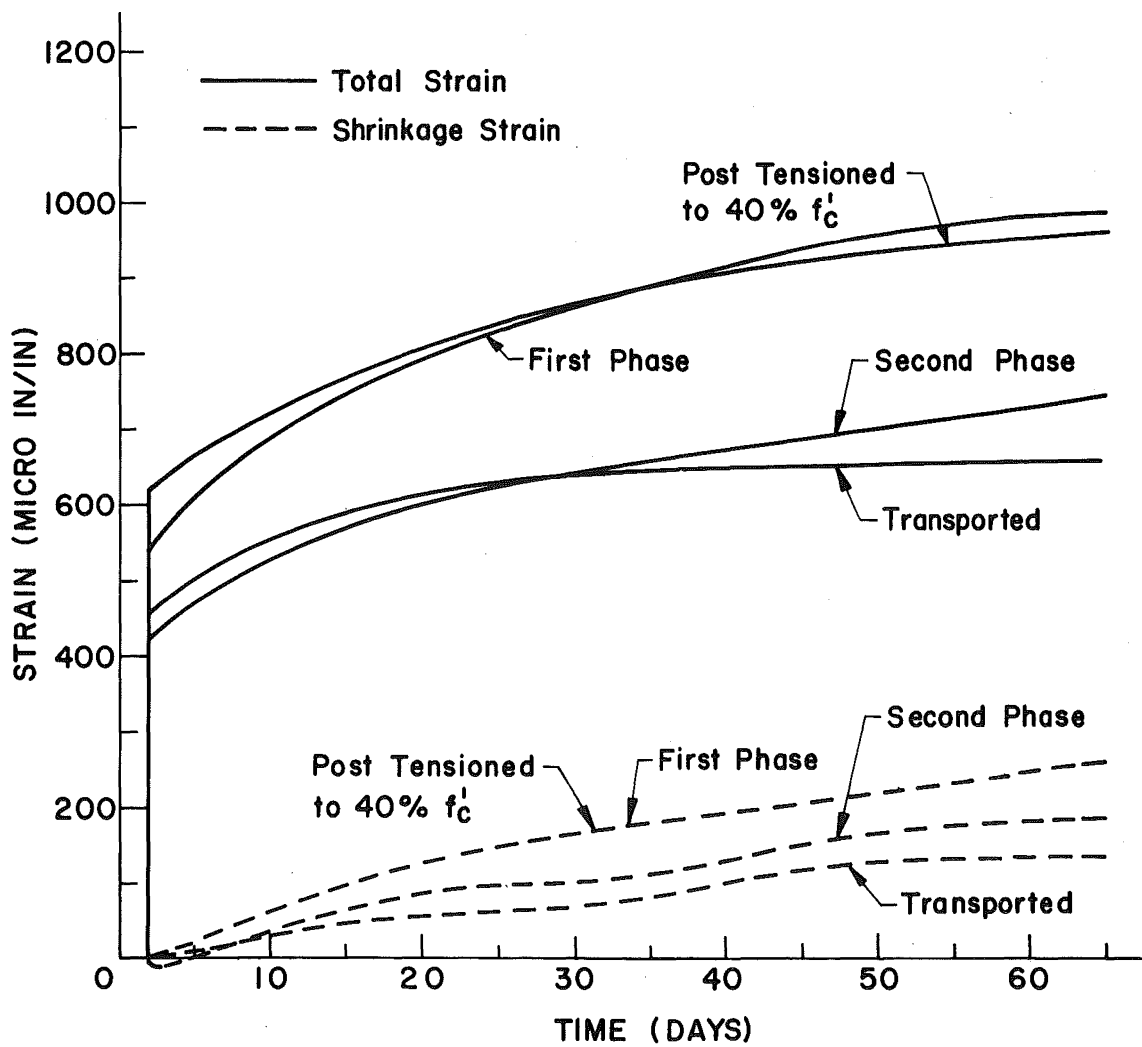


Fig. 14 Total and Shrinkage Strains in Series D

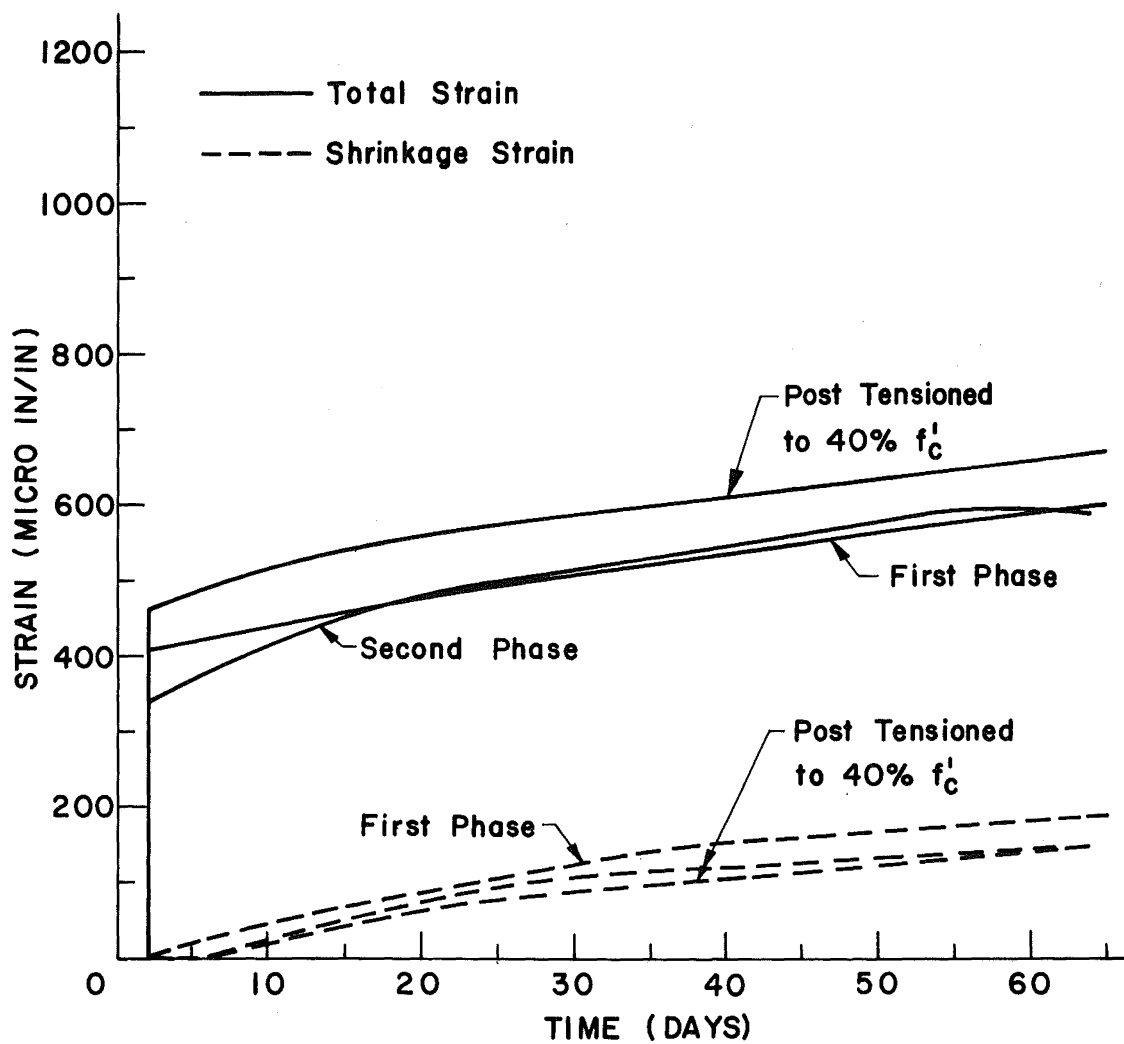


Fig. 15 Total and Shrinkage Strains in Series E

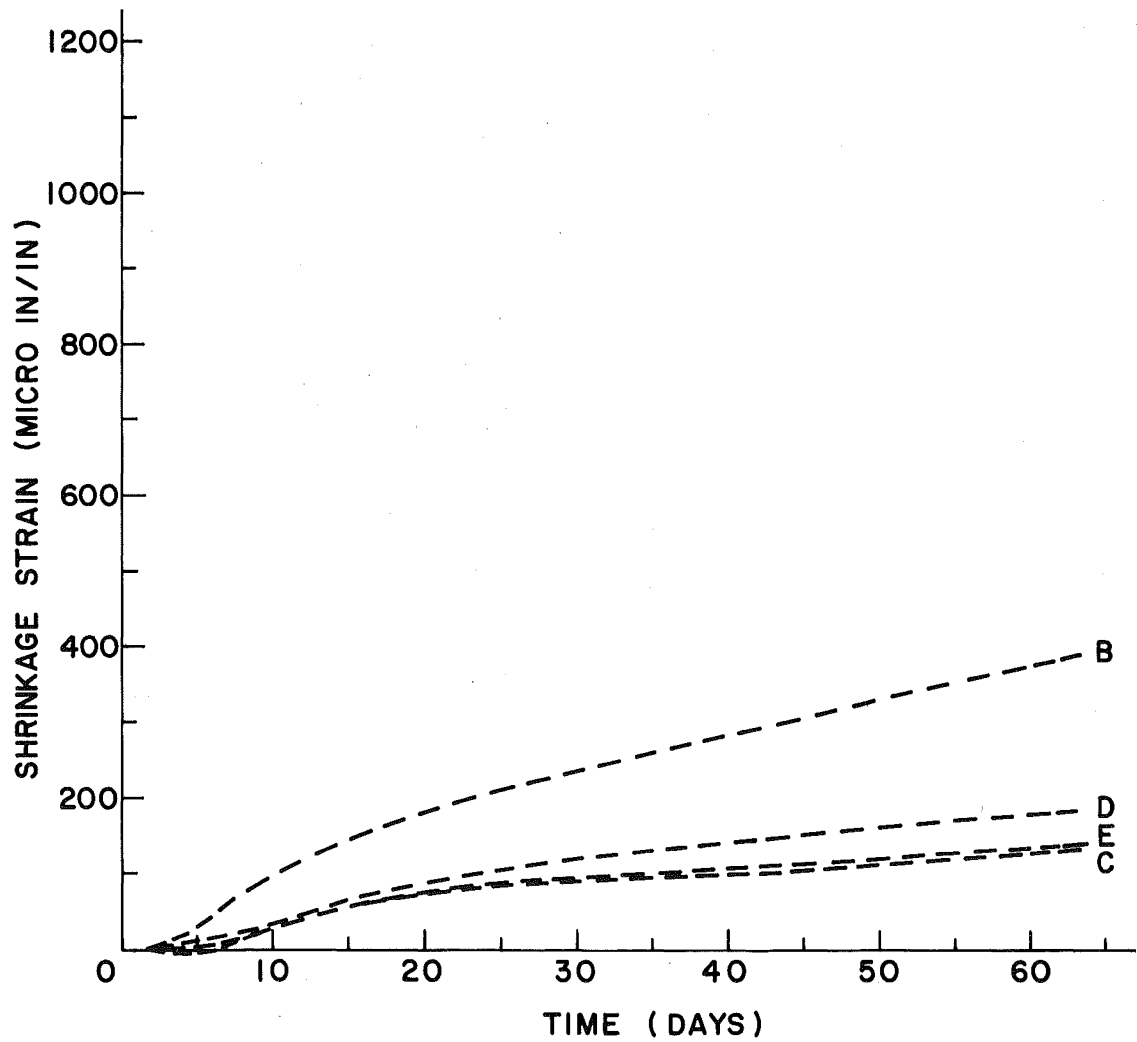


Fig. 16 Comparison of Shrinkage Strains in the Second Phase

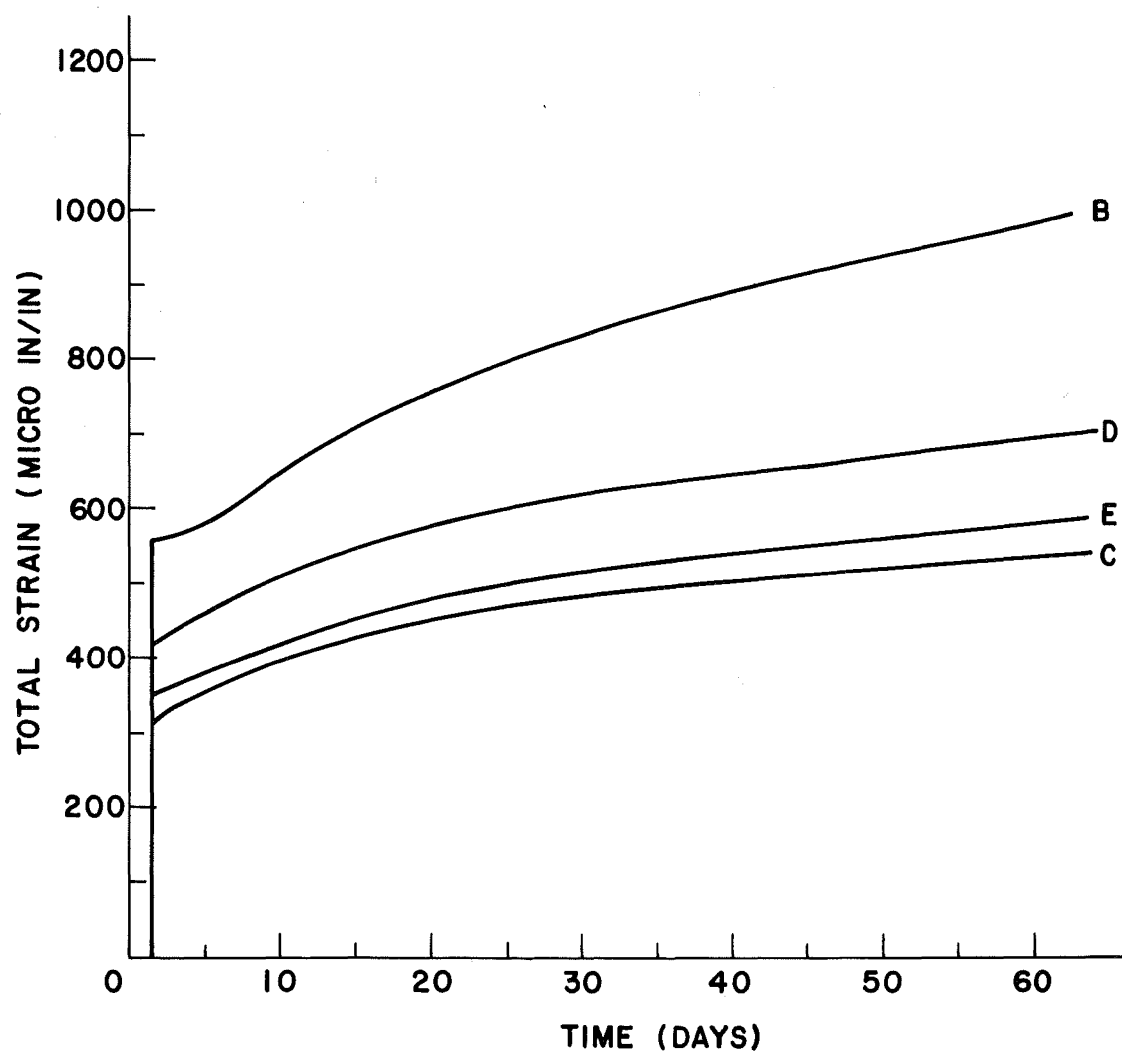


Fig. 17 Comparison of Total Strains in the Second Phase

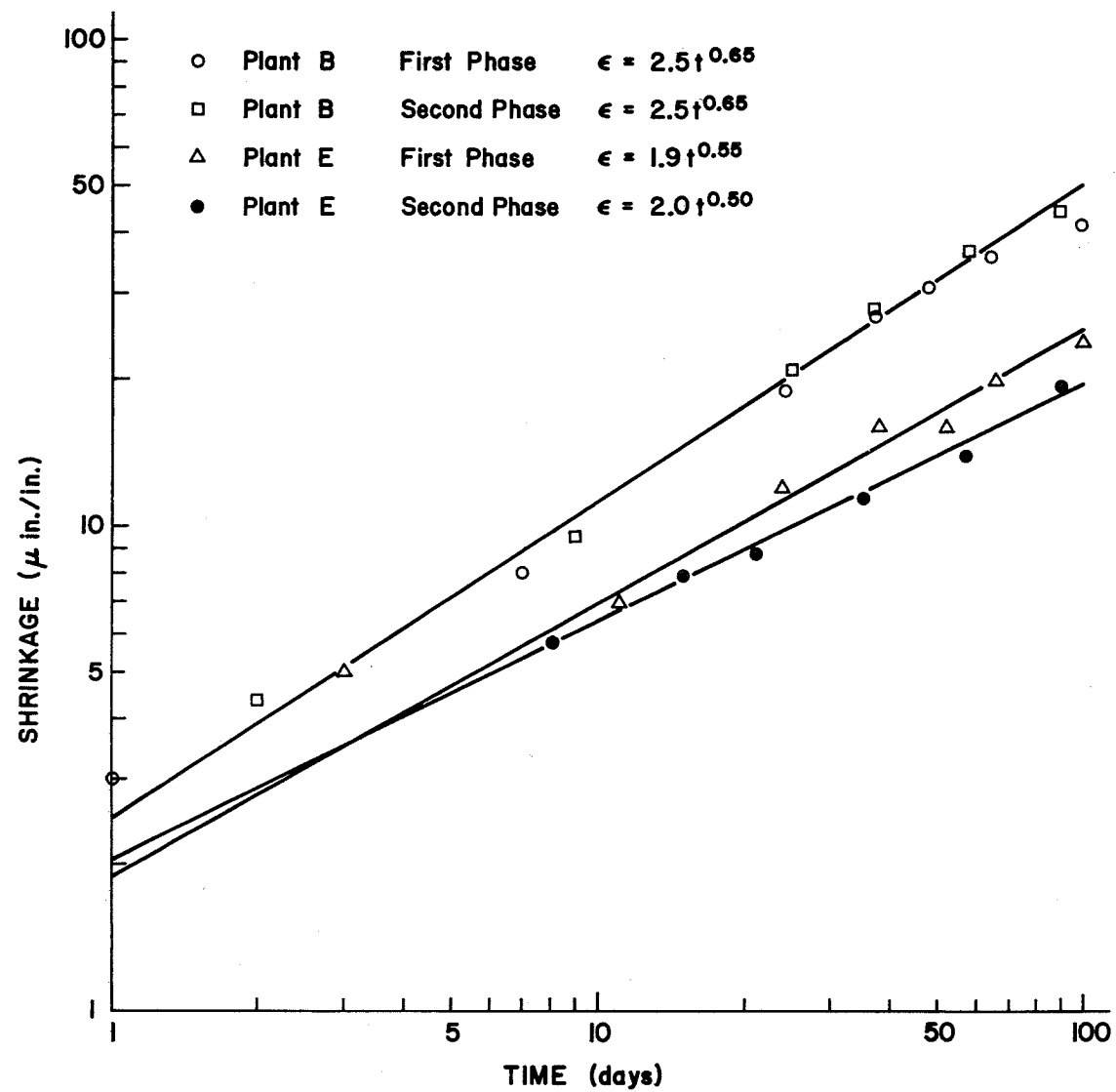


Fig. 18 Shrinkage Strains on Log-Log Scale

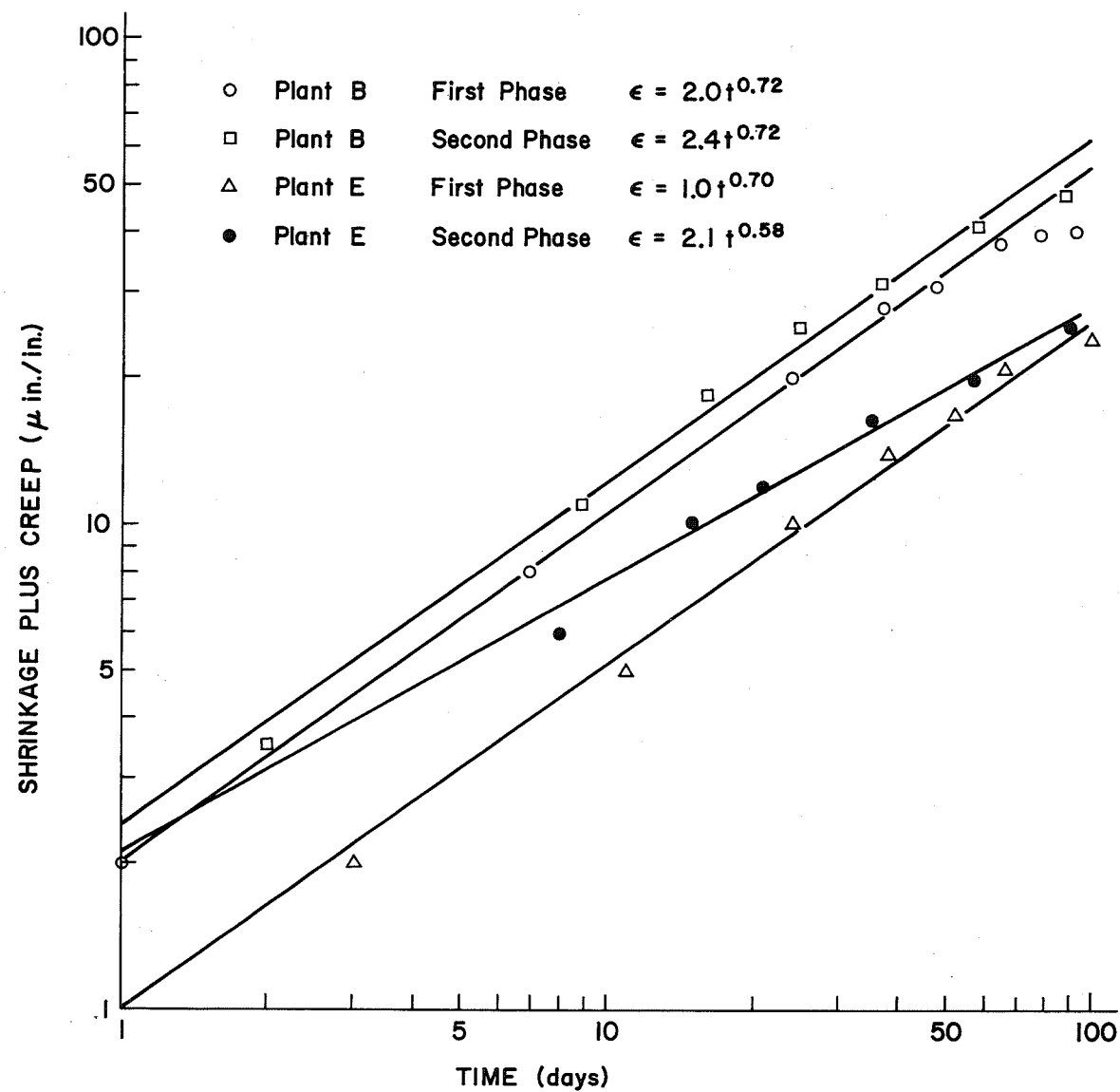


Fig. 19 Creep Strains on Log-Log Scale

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